

Demodulation FM/PM Signals

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Introduction

Frequency demodulators produce output voltage whose instantaneous amplitude is directly proportional to the instantaneous frequency of the input FM wave.

There exist many methods for demodulation of FM signals. For example:

- 1) Frequency Discriminator followed by an envelope detector.
 - a. Slope Circuit
 - i. signal differentiation method
- 2) PLL (Phase locked loop) demodulator.
- 3) Zero-crossing detector.
- 4) Ratio-detector. (old)

Frequency Discriminator (Differentiator)

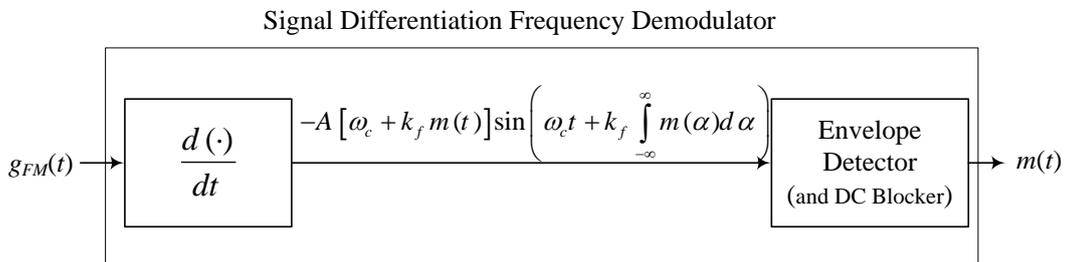
An FM signal has the following form
$$g_{FM}(t) = A \cos \left(\omega_c t + k_f \int_{-\infty}^{\infty} m(\alpha) d\alpha \right).$$

So, its magnitude is constant with value of A . The information is contained in the frequency (or angle) of the FM signal. To extract the message signal contained in an FM signal, we can transfer the information from the angle to the magnitude by simply differentiating the FM signal. Since the derivate of a sinusoid results in multiplying the magnitude of the sinusoid by the derivate of its angle, the derivative of the above FM signal becomes



$$\frac{dg_{FM}(t)}{dt} = -A [\omega_c + k_f m(t)] \sin \left(\omega_c t + k_f \int_{-\infty}^{\infty} m(\alpha) d\alpha \right).$$

So, the message signal of the above derivative is contained in the frequency of the sinusoid and also in its magnitude. It is AM + FM signal. Passing the derivative of the FM signal through an envelope detector will give the desired message signal at the output. Therefore, the following block diagram is an FM demodulator.



Envelope detector can be used because $\Delta\omega = k_f m_p < \omega_c$ and $\Delta\omega + k_f m_p > 0$. The message is always represented by the positive term of the envelope.

Extension to Phase Modulation (PM)

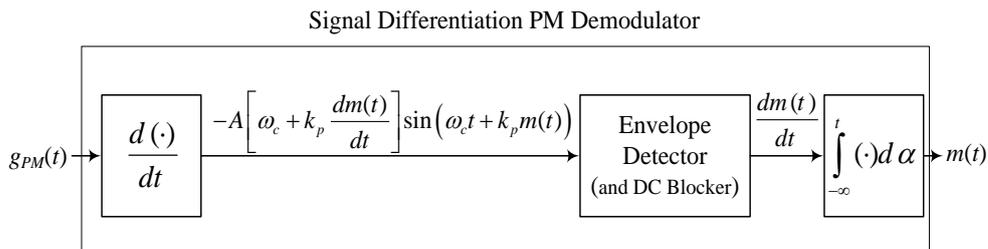
The same idea can be used for PM demodulation. A PM signal has the form

$$g_{PM}(t) = A \cos(\omega_c t + k_p m(t)).$$

So, if we differentiate it, we get

$$\frac{dg_{PM}(t)}{dt} = -A \left[\omega_c + k_p \frac{dm(t)}{dt} \right] \sin(\omega_c t + k_p m(t)).$$

If this signal is passed through an envelope detector, the output will be proportional to the derivative of the message signal. Passing this signal through an integrator will give us what we want. Therefore, the block diagram of a PM demodulator will be as follows.

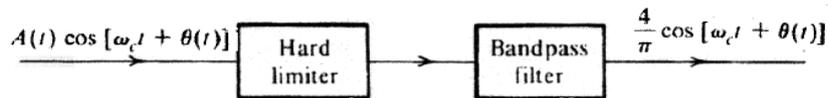


Band-pass Limiter

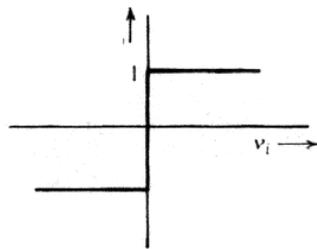
What if the amplitude A is not constant because of channel noise? i.e. $A(t)$

At the output of the differentiator, we will have other terms $\frac{dA(t)}{dt}$.

The bandpass limiter is used to eliminate amplitude variations. **The bandpass limiter consists of a hard-limiter + a bandpass filter.**

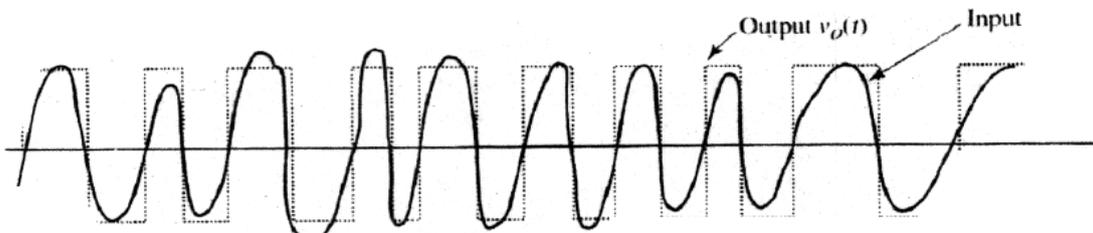


The output/input relation for the hard limiter is shown in the figure below



$$v_o(\theta) = \begin{cases} 1 & \cos\theta > 0 \\ -1 & \cos\theta < 0 \end{cases}$$

In case of amplitude variation the modulated signal, $v_i(t) = A(t)\cos(\theta(t))$ where $\theta(t) = \omega_c t + k_f a(t)$, will appear at the output of the hard-limiter as constant amplitude angle modulated square wave, $v_o(t)$.



The square wave can be represented as sum of sinusoids,

$$v_o(\theta) = \frac{4}{\pi} \left(\cos\theta - \frac{1}{3}\cos(3\theta) + \frac{1}{5}\cos(5\theta) + \dots \right).$$

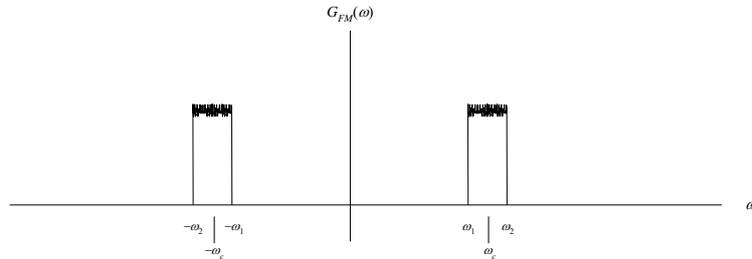
Using a bandpass filter

$$e_o(\theta) = \frac{4}{\pi} (\cos\theta) = \frac{4}{\pi} \left(\cos \left(\omega_c t + k_f \int m(\alpha) d\alpha \right) \right)$$

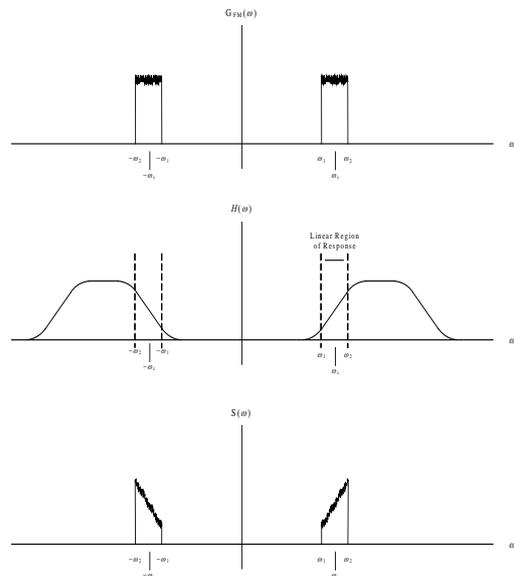
The use of bandpass limiter also suppresses the channel noise when the noise is small.

Generalizing the signal differentiation to any frequency discrimination method

The same concept as described above in the signal differentiation method for demodulation FM and PM signals can be used but after replacing the derivative block at the beginning with a filter. Assume that we would like to demodulate the following FM signal which has a carrier frequency of ω_c and a frequency band from ω_1 to ω_2 .

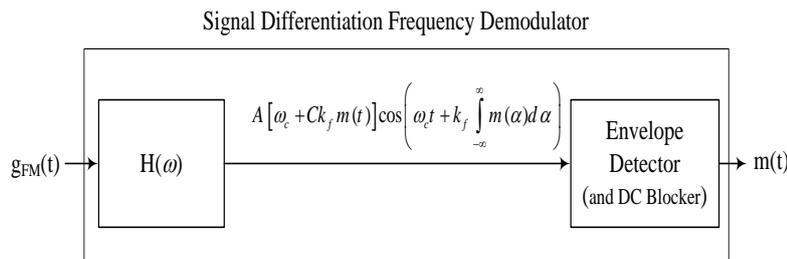


We can use the following BPF which is centered not at ω_c but at a higher frequency such that the range of frequency $[\omega_1, \omega_2]$ falls in the transition band of the filter (in the region where the filter changes from not passing to passing the input signal). If the transition band of the filter has a linear response (a line with some non-zero slope), the different parts of the FM signal, which is input to the filter, will be amplified (or attenuated) by different factors depending on the frequency of these parts. The higher the frequency, the higher the amplitude of the output signal of the filter, and the lower the frequency, the lower the amplitude of the output signal. This process is very similar to what the differentiator in the signal differentiation FM demodulation method does.



Therefore, passing the signal $s(t)$ that is outputted by the BPF into an envelope detector gives a signal that is proportional to the message signal.

Therefore the following block diagram is the frequency discrimination FM demodulator.



Phase-locked Loops

One of these methods is phase-locked loops (PLL) that were studied in a previously. PLLs when fed with an FM signal directly produce an output signal that is proportional to the message signal. PLL has low cost and superior performance even at low SNR (signal-to-noise ratio)

Where do we take the output? Compare with the case of carrier acquisition.

