- Any sinusoidal (cos) signal can be completely specified by its amplitude and angle  $x(t) = A(t)\cos(\omega_c t + \varphi(t))$
- The angle of the cosine  $\theta(t) = \omega_c t + \varphi(t)$
- The **Instantaneous frequency** is given by  $\omega_i(t) = \frac{d\theta(t)}{dt}$ .
- **Phase Modulation (PM)**: the modulated signal have the form  $g_{PM}(t) = A \cdot \cos\left[\omega_c t + k_p m(t)\right],$

The phase and instantaneous frequency of this signal are  $\theta_{PM}(t) = \omega_c t + k_p m(t)$ ,

$$\omega_{i}(t) = \omega_{c} + k_{p} \frac{dm(t)}{dt} = \omega_{c} + k_{p} \dot{m}(t).$$

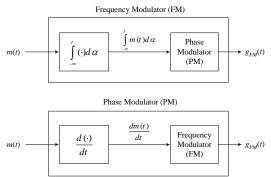
• The frequency Modulation (FM): the modulated signal have the form

$$g_{FM}(t) = A \cdot \cos \left[ \omega_c t + k_f \int_{-\infty}^t m(\alpha) d\alpha \right]$$

The phase and instantaneous frequency of this signal are

$$\theta_{FM}(t) = \omega_c t + k_f \int_{-\infty}^{t} m(\alpha) d\alpha,$$
  
$$\omega_i(t) = \omega_c + k_f \frac{d}{dt} \left[ \int_{-\infty}^{t} m(\alpha) d\alpha \right] = \omega_c + k_f m(t).$$

- Given a signal m(t) you should be able to sketch the FM and PM Modulated signals. See Examples 5.1 & 5.2, notice the continuity issue.
- Relation between PM and FM



## Bandwidth of Angle Modulated signals

In terms of Bandwidth FM/PM can be classified into Wideband and Narrowband. The condition for Narrowband is  $k_f a(t) \ll 1$  for FM and  $k_p m(t) \ll 1$  for the PM case. Note that

$$a(t) = \int_{-\infty}^{t} m(\alpha) d\alpha$$

In general FM can also be written as

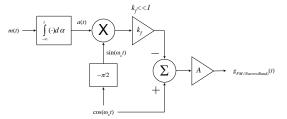
$$g_{FM}(t) = \operatorname{Re}\left\{\hat{g}_{FM}(t)\right\}$$
  
=  $A \cdot \left[\cos(\omega_{c}t) - k_{f}a(t)\sin(\omega_{c}t) - \frac{k_{f}^{2}a^{2}(t)}{2!}\cos(\omega_{c}t) + \frac{k_{f}^{3}a^{3}(t)}{3!}\sin(\omega_{c}t) + \frac{k_{f}^{4}a^{4}(t)}{4!}\cos(\omega_{c}t) + \dots\right]$ 

For the case of narrow band

 $g_{FM(Narrowband)}(t) \approx A \cdot \left[\cos(\omega_c t) - k_f a(t) \sin(\omega_c t)\right]$  $g_{PM(Narrowband)}(t) \approx A \cdot \left[\cos(\omega_c t) - k_p m(t) \sin(\omega_c t)\right]$ 

Like DSB+C (with some difference). Therefore, the bandwidth of the narrowband FM signal is approximately 2B, where B is the bandwidth of the message.

# **Generation/Construction of Narrowband Frequency and Phase Modulators**



Beta is the modulation index (frequency deviation ratio):  $\beta = \frac{\Delta f}{R}$ 

$$\Delta f_{FM} = \frac{k_f m_p}{2\pi}$$
,  $\Delta f_{PM} = \frac{k_p \dot{m}_p}{2\pi}$ ,  $m_p$  is the maximum negative peak of  $m(t)$ , and  $\dot{m}_p$  is the

maximum negative peak of the derivative of m(t)

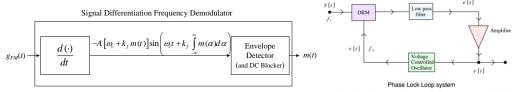
Carson's Rule 
$$BW = 2(\Delta f + B) = 2B(\beta + 1)$$

See Examples 5.3, 5.4, 5.5

- If FM is not efficient in terms of BW compared with AM, Why FM? (mention three reasons)
- Analysis of performance of FM signals under nonlinearity
- Generation of Wideband FM (WBFM):
  - Direct Method VCO (+,-)
  - Indirect Armstrong Method (See Examples)

# **Demodulation of FM/PM signals**

- 1) Frequency Discriminator
- 2) Phase Locked Loop (PLL)
- 3) Zero Crossing Detector
- 4) Ratio Detector
- Sketch how their block diagram, how they work, Advantages and disadvantages of each.
- A bandpss limiter which eliminates amplitude variations is a hard limiter followed by a BPF.



## DBM: Double balanced modulator (multiplier)

#### **FM Receiver**

88-108 MHz, 200 kHz/ Channel, IF=10.7 MHz

Super-heterodyne concept with envelope detector replaced by PLL or frequency discriminator. Monophonic & compatibility with stereophonic (1dB). *Fully understand Figure 5.18*, 5<sup>th</sup> Ed.

#### Note:

- Spectral Analysis of Tone Frequency Modulation (Bessel Functions) is not required. (p 214-216 5th ed.)
- PLL (error analysis is not included p233-234, 5<sup>th</sup> ed.)

This summary prepared by *Dr. Ali Muqaibel* is meant to help you visualize the main ideas. It is not meant to replace the book. If you have any comment or correction, please contact Dr. Ali Muqaibel