EE 370 991

Solution of HW Set #7

6.1-1: The Nyquist sampling frequency (rate) is $f_N = 2B$ Hz and the interval is $T_{SN} = \frac{1}{f_N} = \frac{1}{2B}$ s, where B is the highest frequency in the signal spectrum.

i) For $g_1(t)$, $B = 10^5$ Hz. Hence, $f_N = 200$ kHz and $T_{SN} = \frac{1}{2\times10^5} = 5\ \mu s$.

ii) For $g_1^2(t)$, $B = 2\times10^5$ Hz. Hence, $f_N = 400$ kHz and $T_{SN} = 2.5\ \mu s$.

iii) For $g_2^3(t)$, $B = 3(1.5\times10^5) = 450$ kHz. Hence, $f_N = 900$ kHz and $T_{SN} = 1.111\ \mu s$.

iv) For $g_1(t)g_2(t)$, $B = 10^5 + 1.5\times10^5 = 2.5\times10^5$ Hz. Hence, $f_N = 500$ kHz and $T_{SN} = 2\ \mu s$

6.1-2: (a) $g(t) = \text{sinc}(100\pi t)$

From Table 3.1, we have

$$\frac{W}{\pi}\,\text{sinc}(Wt) \longleftrightarrow \text{rect}\left(\frac{\omega}{2W}\right)$$

Therefore, $G(\omega) = \frac{\pi}{100\pi}\,\text{rect}\left(\frac{\omega}{200\pi}\right)$, and $B = \frac{200\pi}{2(2\pi)} = 50$ Hz.

We conclude that $f_N = 100$ Hz and $T_{SN} = \frac{1}{100} = 10$ ms.

(b) $g(t) = \text{sinc}^2(100\pi t)$

$$G(\omega) = \frac{1}{2\pi}\left\{\frac{\pi}{100\pi}\,\text{rect}\left(\frac{\omega}{200\pi}\right) * \frac{\pi}{100\pi}\,\text{rect}\left(\frac{\omega}{200\pi}\right)\right\}$$

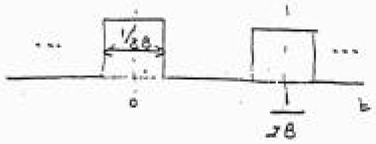
Hence, $B = 2\left(\frac{200\pi}{4\pi}\right) = 100$ Hz, $f_N = 200$ Hz, and $T_{SN} = \frac{1}{200} = 5$ ms.

(c) $g(t) = \text{sinc}(100\pi t) + \text{sinc}(50\pi t)$

$$G(\omega) = \frac{\pi}{100\pi}\,\text{rect}\left(\frac{\omega}{200\pi}\right) + \frac{\pi}{50\pi}\,\text{rect}\left(\frac{\omega}{100\pi}\right)$$

Hence, $B = 50$ Hz, $f_N = 100$ Hz, and $T_{SN} = 10$ ms as in part (a) above.

6.1-3: $p_{T_s}(t) = \sum_{n=-\infty}^{\infty} \text{rect}\left(\frac{t-nT_s}{1/8B}\right)$



$$\bar{g}(t) = g(t) \cdot \sum_{n=-\infty}^{\infty} \text{rect}\left(\frac{t-nT_s}{1/8B}\right)$$

$p_{T_s}(t)$ is periodic in time with period $T = \frac{1}{2B}$, therefore, we can expand $p_{T_s}(t)$ in Fourier series of the form

$$p_{T_s}(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t \quad , \omega_0 = 4\pi B = \omega_s.$$

$$a_0 = 2B \int_{-1/16B}^{1/16B} dt = 2B\left(\frac{1}{8B}\right) = \frac{1}{4}.$$

$$a_n = 4B \int_{-1/16B}^{1/16B} \cos 4\pi n B t \, dt = \frac{4B}{4\pi n B} 2 \sin \frac{n\pi}{4}$$

$$= \frac{2}{\pi n} \sin(n\pi/4)$$

$$b_n = 4B \int_{-1/16B}^{1/16B} \sin 4\pi n B t \, dt = 0.$$

Substituting for $p_{T_s}(t)$, we obtain

$$\bar{g}(t) = \frac{1}{4} g(t) + \sum_{n=1}^{\infty} \frac{2}{\pi n} \sin\left(\frac{n\pi}{4}\right) g(t) \cos n\omega_s t \quad , \omega_s = 4\pi B.$$

By passing $\bar{g}(t)$ through an ideal low-pass filter of bandwidth B Hz and gain of 4, we obtain

$$G_0(\omega) = \frac{1}{4} G(\omega) \cdot 4 \,\text{rect}\left(\frac{\omega}{4\pi B}\right) = G(\omega).$$

Take the inverse Fourier transform, to obtain

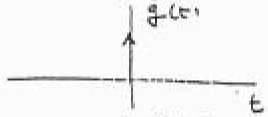
$$g_0(t) = g(t).$$

<u>6.1-4:</u> $g(t) = \text{sinc}^2(5\pi t)$

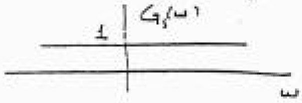
(i) For sampling rate of $5\ H_z$, $T_s = \frac{1}{5}$ s.

$$g(nT_s) = \text{sinc}^2\left(\frac{5\pi n}{5}\right) = \text{sinc}^2(n\pi) = \left(\frac{\sin n\pi}{n\pi}\right)^2 = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0. \end{cases}$$

a) $\bar{g}(t) = \delta(t)$



b) $G_s(\omega) = 1$



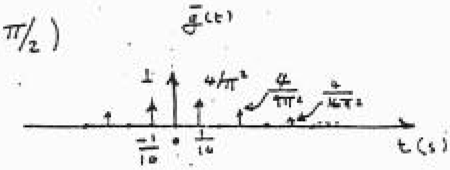
c) You can not recover $g(t)$ from $\bar{g}(t)$ $\left(f_s = 5H_z,\ B = \frac{10\pi}{2\pi} = 5\ H_z \right)$

d) $G_0(\omega) = \text{rect}\left(\frac{\omega}{20\pi}\right)$

(ii) For a sampling rate of $10\ H_z$, $T_s = \frac{1}{10}$ s.

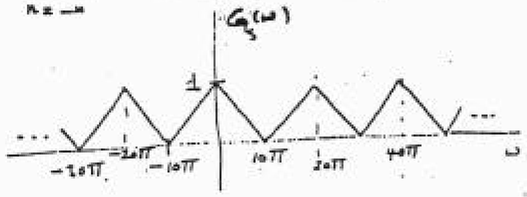
$$g(nT_s) = \text{sinc}^2\left(\frac{5\pi n}{10}\right) = \text{sinc}^2(n\pi/2)$$

a) $\bar{g}(t) = \sum_{n=-\infty}^{\infty} \text{sinc}^2\left(\frac{n\pi}{2}\right) \delta\left(t - \frac{n}{10}\right)$



b) $G_s(\omega) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} G(\omega - n\omega_s) = 10 \sum_{n=-\infty}^{\infty} G(\omega - n\omega_s)$, $\omega_s = 20\pi$

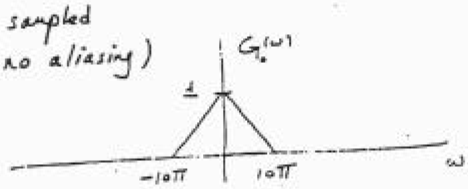
$$G(\omega) = \frac{2\pi}{10\pi}\, \Delta\left(\frac{\omega}{20\pi}\right)$$
$$= \frac{1}{10}\, \Delta\left(\frac{\omega}{20\pi}\right)$$



c) You can recover $g(t)$ from the sampled signal because $\omega_s = 4\pi B = 20\pi$ (no aliasing)

d) $G_0(\omega) = \Delta\left(\frac{\omega}{20\pi}\right)$

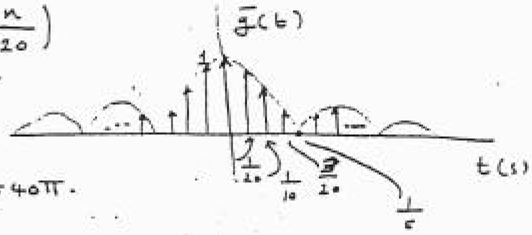
$g_0(t) = 10\ \text{sinc}^2(5\pi t)$



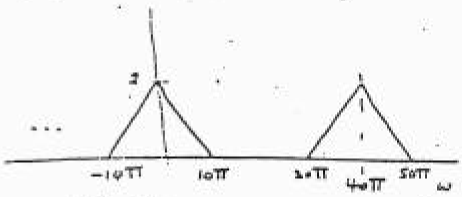
iii) For a sampling rate of 20 Hz, $T_s = \frac{1}{20}$ s.

$$g(nT_s) = \text{sinc}^2\left(\frac{5\pi n}{20}\right) = \text{sinc}^2\left(\frac{n\pi}{4}\right)$$

a) $$\bar{g}(t) = \sum_{n=-\infty}^{\infty} \text{sinc}^2\left(\frac{n\pi}{4}\right)\delta\left(t - \frac{n}{20}\right)$$

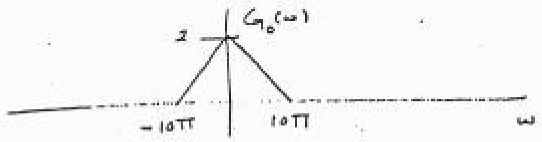


b) $$G_s(\omega) = 20 \sum_{n=-\infty}^{\infty} G(\omega - n\omega_s), \quad \omega_s = 40\pi.$$



c) You can recover $g(t)$ since $\omega_s > 20\pi$.

d) $$G_0(\omega) = 2\,\Delta\left(\frac{\omega}{20\pi}\right)$$
$$g_0(t) = 20\,\text{sinc}^2(5\pi t)$$



6.1-5: $g_1(t) = 10^4\,\text{rect}(10^4 t)$ & $g_2(t) = \delta(t)$

$H_1(\omega)$ has a bandwidth equal to $20{,}000\pi$ rad/s or $B_1 = 10$ kHz.

$H_2(\omega)$ has a bandwidth equal to 5 kHz.

Therefore, the bandwidths of $y_1(t)$ and $y_2(t)$ are 10 kHz and 5 kHz, respectively. The bandwidth of the product $y_1(t)y_2(t)$ is the sum of the individual bandwidth and is equal to 15 kHz. The Nyquist rate of $y_1(t)$, $y_2(t)$, and $y(t)$ is 20 kHz, 10 kHz, and 30 kHz, respectively.