

S.1-1:

FM: $\Delta\omega = k_f m_p = 10^5$

$$\omega_i = \omega_c + k_f m(t)$$

$$\omega_{\max} = \omega_c + k_f = 10^8 + 10^5 \text{ rad/s}$$

$$\omega_{\min} = \omega_c - k_f = 10^8 - 10^5 \text{ rad/s}$$

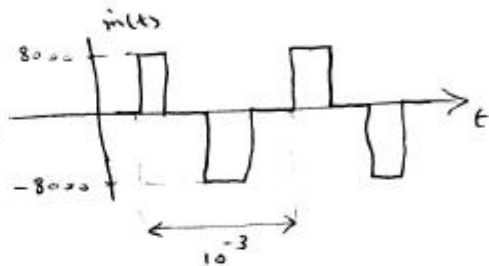
PM: $\Delta\omega = k_p m'_p = 25(8000)$
 $= 2 \times 10^5 \text{ rad/s}$

$$\omega_i = \omega_c + k_p m(t)$$

$$\omega_{\max} = \omega_c + 25(8000)$$

$$= 10^8 + 2 \times 10^5 \text{ rad/s}$$

$$\omega_{\min} = 10^8 - 2 \times 10^5 \text{ rad/s}$$

S.1-2:

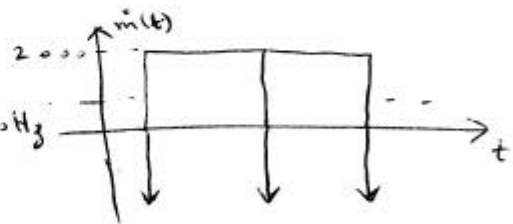
FM: $\Delta f = \frac{k_f}{2\pi} m_p = 1000 \text{ Hz}$

$$f_i = f_c + \frac{k_f}{2\pi} m(t)$$

$$f_{\max} = f_c + \frac{k_f}{2\pi} (1) = 10^6 + 1000 \text{ Hz}$$

$$f_{\min} = 10^6 - 1000 \text{ Hz}$$

PM: $\Delta f = \frac{k_p}{2\pi} m'_p = \frac{2000}{4} = 500 \text{ Hz}$



$$f_i = f_c + \frac{k_p}{2\pi} m(t)$$

$$\theta_i = 2\pi f_c t + k_p m(t)$$

$$\theta_{\max} = 2\pi f_c t + \frac{\pi}{2} (1) = 2\pi \times 10^6 t + \frac{\pi}{2} \text{ rad}$$

$$\theta_{\min} = 2\pi \times 10^6 t - \frac{\pi}{2} \text{ rad}$$

So, phase of the PM signal changes between θ_{\min} & θ_{\max}

If $k_p \geq \pi$, $\theta_{\max} = 2\pi \times 10^6 t + k_p$

$$\theta_{\min} = 2\pi \times 10^6 t - k_p$$

So, the range $(\theta_{\min}, \theta_{\max})$ spans the unit circle more than one time \Rightarrow different input value \Rightarrow same phase.

S.2-1:

$$m(t) = 2 \cos 100t + 18 \cos 2000\pi t$$

a) FM:

$$\begin{aligned} \varphi_{FM}(t) &= A \cos \left[\omega_c t + k_f \int_{-\infty}^t m(\tau) d\tau \right] \\ &= 10 \cos \left[10^6 t + 1000\pi \left(\frac{1}{50} \sin 100t + \frac{9}{1200\pi} \sin 2000\pi t \right) \right] \\ &= 10 \cos \left[10^6 t + 20\pi \sin 100t + 9 \sin 2000\pi t \right] \end{aligned}$$

PM:

$$\begin{aligned} \varphi_{PM}(t) &= A \cos \left[\omega_c t + k_p m(t) \right] \\ &= 10 \cos \left[10^6 t + 2 \cos 100t + 18 \cos 2000\pi t \right] \end{aligned}$$

b) FM: $\Delta\omega = k_f m_p = 1000\pi (2 + 18) = 2 \times 10^4 \pi$

$$\begin{aligned} \Rightarrow BW_{FM} &= 2 \left(\frac{\Delta\omega}{2\pi} + B \right) \\ &= 2 \left(10^4 + 10^3 \right) = 21000 \text{ Hz} \end{aligned}$$

PM: $\Delta\omega = k_p m_p' = 200 + 18(2000\pi) = 36200\pi$

$$\begin{aligned} \Rightarrow BW_{PM} &= 2 \left(\frac{\Delta\omega}{2\pi} + B \right) \\ &= 2 \left(18100 + 10^3 \right) = 38200 \text{ Hz} \end{aligned}$$

S.2-2:

$$\varphi_{EM}(t) = 10 \cos(\omega_c t + 0.1 \sin 2000\pi t)$$

a) Power = $\frac{10^2}{2} = 50 \text{ W}$

b) Δf is found from the instantaneous frequency

$$f_i = f_c + \frac{1}{2\pi} 2000\pi \cos(2000\pi t)$$

$$\Rightarrow k_f = 1000 \text{ Hz} \quad \Rightarrow \Delta f = k_f m_p = 100 \text{ Hz}$$

c) $\Delta\phi = k_p m_p = 0.1 \text{ rad}$

d) $BW = 2(\Delta f + B) = 2(100 + 1000) = 2.2 \text{ KHz}$