3.1-4 (b) :

$$g(t) = e^{a(t-T)}, \quad 0 \leq t \leq T$$

$$G(\omega) = \int_0^T e^{a(t-T)} e^{-j\omega t} \, dt$$

$$= e^{-aT} \int_0^T e^{t(a-j\omega)} \, dt$$

$$= \frac{e^{-aT}}{a-j\omega} \left[e^{(a-j\omega)T} - 1 \right]$$

OR: By applying the Time-Shifting property:

Let $v(t) = e^{at}, \quad -T \leq t \leq 0$

$$\Rightarrow \quad g(t) = v(t-T)$$

$$\Rightarrow \quad G(\omega) = V(\omega) \, e^{-j\omega T}$$

$$V(\omega) = \int_{-T}^{0} e^{at} e^{-j\omega t} \, dt$$

$$= \frac{1}{a-j\omega} \left[1 - e^{-T(a-j\omega)} \right]$$

$$\Rightarrow \quad G(\omega) = \frac{e^{-j\omega T}}{a-j\omega} \left[1 - e^{-T(a-j\omega)} \right]$$

$$= \frac{e^{-aT}}{a-j\omega} \left[e^{(a-j\omega)T} - 1 \right]$$

Note that the two methods have the same answer!

3.1-7(a)

$$g(t) = \frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\pi/2} \cos w \, e^{jwt}\, dw = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} \frac{e^{jw}+e^{-jw}}{2} e^{jwt}\, dw$$

$$= \frac{1}{4\pi}\int_{-\frac{\pi}{2}}^{\pi/2}\left[e^{j(1+t)w} + e^{-j(1-t)w}\right] dw$$

$$= \frac{1}{4\pi}\left[\left.\frac{e^{j(1+t)w}}{j(1+t)}\right|_{-\pi/2}^{\pi/2} + \left.\frac{e^{-j(1-t)w}}{-j(1-t)}\right|_{-\pi/2}^{\pi/2}\right]$$

$$= \frac{1}{4\pi}\left[\frac{e^{j(1+t)\pi/2} - e^{-j(1+t)\frac{\pi}{2}}}{j(1+t)} + \frac{e^{j(1-t)\frac{\pi}{2}} - e^{-j(1-t)\frac{\pi}{2}}}{j(1-t)}\right]$$

$$= \frac{1}{4\pi}\left[\frac{je^{j\pi t/2} + je^{-j\pi t/2}}{j(1+t)} + \frac{je^{-j\pi t/2} + je^{j\pi t/2}}{j(1-t)}\right]$$

$$= \frac{1}{4\pi}\left[\frac{\cos \pi t/2}{\frac{1+t}{2}} + \frac{\cos \pi t/2}{\frac{1-t}{2}}\right]$$

$$= \frac{1}{4\pi}\left[\frac{2}{1+t} + \frac{2}{1-t}\right]\cos\frac{\pi t}{2} = \frac{1}{\pi(1-t^2)}\cos \pi t/2$$

3.3-6(a) $g(t) = \Delta\left(\frac{t}{2\pi}\right)\cos 10t$ where $\Delta\left(\frac{t}{2\pi}\right)$ is triangle function defined in pg 79 of text book.

From table

$$\Delta\left(\frac{t}{2\pi}\right) \longleftrightarrow \pi \,\text{sinc}^2\left(\frac{\pi w}{2}\right)$$

cont

3.3-6(a) (cont.) using modulation property:

$$g(t) = \Delta\left(\frac{t}{2\pi}\right)\cos 10t \Longleftrightarrow \frac{\pi}{2}\left\{\operatorname{sinc}^2\left[\frac{\pi(\omega-10)}{2}\right] + \operatorname{sinc}^2\left[\frac{\pi(\omega+10)}{2}\right]\right\}$$

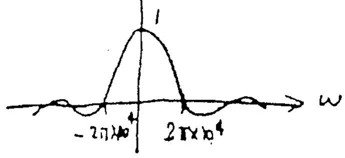
3.3-6(b) $g(t)$ is the same as the signal in Fig (a) delayed by 2π. i.e $g(t) = \Delta\left(\frac{t-2\pi}{2\pi}\right)\cos[10(t-2\pi)]$

Using time shifting property

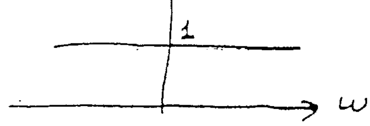
$$G(\omega) = \frac{\pi}{2}\left\{\operatorname{sinc}^2\left[\frac{\pi(\omega-10)}{2}\right] + \operatorname{sinc}^2\left[\frac{\pi(\omega+10)}{2}\right]\right\}e^{-j2\pi\omega}$$

3.4-1 a)

$G_1(\omega) = \operatorname{sinc}\left(\frac{\omega}{2\times 10^4}\right)$

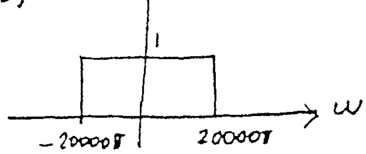


$G_2(\omega) = 1$

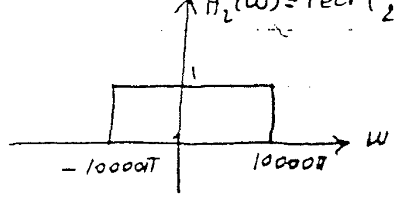


b)

$H_1(\omega) = \operatorname{rect}\left(\frac{\omega}{400000\pi}\right)$

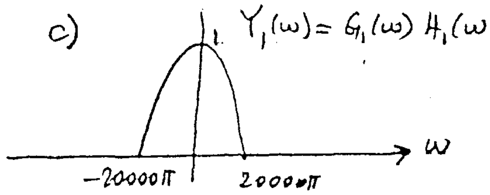


$H_2(\omega) = \operatorname{rect}\left(\frac{\omega}{200000\pi}\right)$

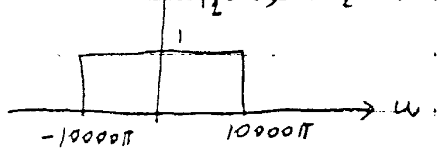


c)

$Y_1(\omega) = G_1(\omega)\,H_1(\omega)$



$Y_2(\omega) = H_2(\omega)$



d) Bandwidth of $y_1(t)$ is 20000π rad/s = 10 KHz.

" " $y_2(t)$ is 10000π rad/s = 5 KHz.

$y(t) = y_1(t)\cdot y_2(t) \Longleftrightarrow Y(\omega) = Y_1(\omega) \otimes Y_2(\omega)$

From property of convolution, bandwidth of $y(t)$ is the sum of bandwidts Y_1 & Y_2 and it is 15kHz.