7.2-1: For full-width rect pulse $p(t) = \text{rect}(t/T_b)$,

$$P(\omega) = T_b \, \text{sinc}\left(\frac{\omega T_b}{2}\right)$$

For polar signaling [see Eq (7.12)]

$$S_y(\omega) = \frac{|P(\omega)|^2}{T_b} = T_b \, \text{sinc}^2\left(\frac{\omega T_b}{2}\right) \quad -(1)$$

For on-off case [see Eq. (7.18b)

$$S_y(\omega) = \frac{|P(\omega)|^2}{4T_b}\left[1 + \frac{2\pi}{T_b}\sum_{n=-\infty}^{\infty}\delta\left(\omega - \frac{2\pi n}{T_b}\right)\right]$$

$$= \frac{T_b}{4}\text{sinc}^2\left(\frac{\omega T_b}{2}\right)\left[1 + \frac{2\pi}{T_b}\sum_{n=-\infty}^{\infty}\delta\left(\omega - \frac{2\pi n}{T_b}\right)\right]$$

But $\text{sinc}^2\left(\frac{\omega T_b}{2}\right) = 0$ for $\omega = \frac{2\pi n}{T_b}$ for all $n \neq 0$, and $= 1$ for $n = 0$. Hence

$$S_y(\omega) = \frac{T_b}{4}\text{sinc}^2\left(\frac{\omega T_b}{2}\right) + \frac{\pi}{2}\delta(\omega)$$

For bipolar case [Eq (7.20b)]

$$S_y(\omega) = \frac{|P(\omega)|^2}{T_b}\sin^2\left(\frac{\omega T_b}{2}\right)$$

$$= T_b \, \text{sinc}^2\left(\frac{\omega T_b}{2}\right)\sin^2\left(\frac{\omega T_b}{2}\right)$$

The PSD's of the three cases are shown in Fig S7.2-1

From these spectra, we find the bandwidths for all

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three cases to be R_b Hz.

The bandwidths for the three cases, when half-width pulses are used, are as follows: Polar and on-off: $2R_b$ Hz; bipolar: R_b Hz. Clearly, for polar and on-off cases the bandwidth is halved when full-width pulses are used. However, for the bipolar case, the bandwidth remains unchanged. The pulse shape has only a minor influence in the bipolar case because the term $\sin^2(\omega T_b/2)$ in $S_y(\omega)$ determines its bandwidth

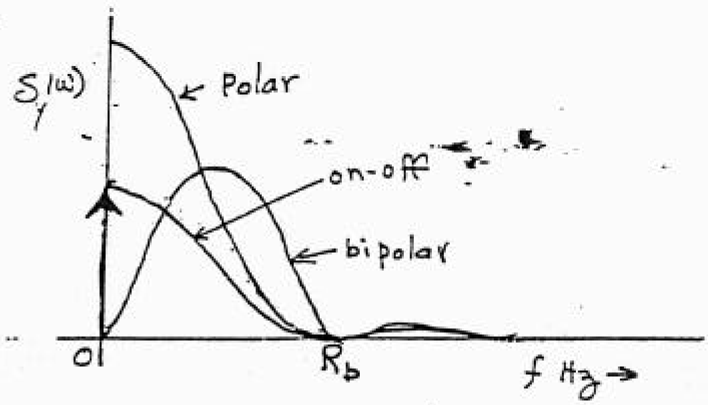

Fig. S7.2-1

7.2-2:

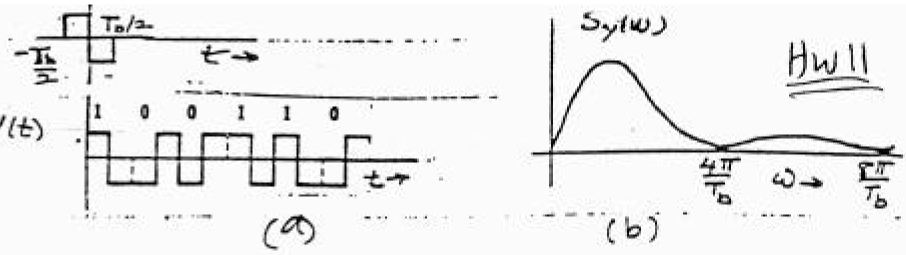

Fig. S7.2-2

$$p(t) = \text{rect}\left(\frac{t+\frac{T_b}{4}}{T_b/2}\right) - \text{rect}\left(\frac{t-\frac{T_b}{4}}{T_b/2}\right)$$

and

$$P(\omega) = \frac{T_b}{2}\,\text{sinc}\left(\frac{\omega T_b}{4}\right) e^{j\omega T_b/4} + \frac{T_b}{2}\,\text{sinc}\left(\frac{\omega T_b}{4}\right) e^{-j\omega T_b/4}$$

$$= j T_b\,\text{sinc}\left(\frac{\omega T_b}{4}\right)\sin\left(\frac{\omega T_b}{4}\right)$$

$$S_y(\omega) = \frac{|P(\omega)|^2}{T_b} = T_b\,\text{sinc}^2\left(\frac{\omega T_b}{4}\right)\sin^2\left(\frac{\omega T_b}{4}\right)$$

From Fig. S7.2-2, it is clear that the bandwidth is $4\pi/T_b$ rad/s or $2R_b$ Hz.

7.2-3: For differential code (Fig. 7.17)

$$R_0 = \lim_{N\to\infty} \frac{1}{N}\left[\frac{N}{2}(1)^2 + \frac{N}{2}(-1)^2\right] = 1$$

To compute R_1, we observe that there are four possible 2-bit sequences 11, 00, 01, and 10, which are equally likely. The product $a_k a_{k+1}$ for the first two combinations is 1 and is -1 for the last two combinations. He

$$R_1 = \lim_{N\to\infty} \frac{1}{N}\left[\frac{N}{2}(1) + \frac{N}{2}(-1)\right] = 0$$

Similarly, we can show that

$$R_n = 0 \qquad n > 1$$

Hence

$$S_y(\omega) = \frac{|P(\omega)|^2}{T_b} = \frac{T_b}{4}\,\text{sinc}^2\left(\frac{\omega T_b}{4}\right)$$