King Fahd University of Petroleum & Minerals
Electrical Engineering Department
EE315: Probabilistic Methods in Electrical Engineering (112)

Major Exam I

March 10, 2012
7:00-8:30 PM
Building 59-Rooms 2001-2004

Name: ___________KEY_______________

ID#_______________________________

Question	Mark
1	/10
2	/10
3	/10
4	/10
Total	/40

Instructions:

1. This is a closed-books/notes exam.
2. The duration of this exam is one and half hours.
3. Read the questions carefully. Plan which question to start with.
4. CLEARLY LABEL ALL SIGNIFICANT VALUES ON BOTH AXIES OF ANY SKETCH
5. Work in your own.
6. Strictly no mobile phones are allowed.
7. Table Attached

Good luck

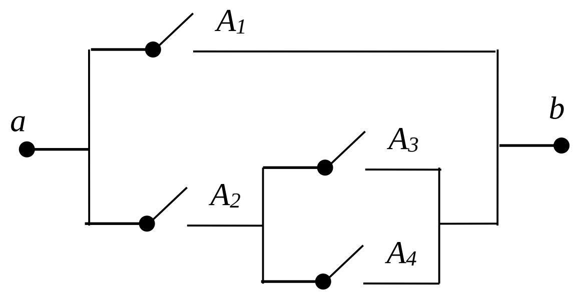
Mark	**ction** sec	**Timing**	**Instructor**
	1	SMW 9:00	Dr. Ahmed Masoud
	2	UT 10:00	Dr. Ali Muqaibel (Coordinator)
	3	UT 08:30	Dr. Saad Al-Ubaidi
	4	UT 10:00	Dr. Saad Al-Ubaidi

<u>Problem 1:</u> (10 points)

Consider the following switching network shown. Let A_1, A_2, A_3, and A_4 denote the events that the associated switches are closed (connecting). Let A_{ab} denote the event that there is a closed path between terminals *a* and *b*. (i.e $A_{ab}closed$)

a) Express A_{ab} in terms of A_1, A_2, A_3, and A_4 **(2 points)**

$$\boldsymbol{A_{ab} = A_1 \cup (A_2 \cap (A_3 \cup A_4)}$$



b) If all switches are independent and the probability of being closed is 0.5. That is $P(A_1) = P(A_2) = P(A_3) = P(A_4) = 0.5$. Find $P(A_{ab})$. *i.e P(path between a and b is closed (connecting))* **(4 points)**

$$\begin{aligned} P(A) &= P[A_1 \cup (A_2 \cap A_3) \cup (A_2 \cap A_4)] \\ &= P(A_1) + P(A_2 \cap A_3) + P(A_2 \cap A_4) \\ &\quad - P[A_1 \cap (A_2 \cap A_3)] - P[A_1 \cap (A_2 \cap A_4)] - P[(A_2 \cap A_3) \cap (A_2 \cap A_4)] \\ &\quad + P[A_1 \cap (A_2 \cap A_3) \cap (A_2 \cap A_4)] \\ &= P(A_1) + P(A_2 \cap A_3) + P(A_2 \cap A_4) \\ &\quad - P(A_1 \cap A_2 \cap A_3) - P(A_1 \cap A_2 \cap A_4) - P(A_2 \cap A_3 \cap A_4) \\ &\quad + P(A_1 \cap A_2 \cap A_3 \cap A_4) \end{aligned}$$

$P(A_{ab}) = 0.5 + 0.5^2 + 0.5^2 - 3(0.5^3) + 0.5^4$=0.6875

It can also be solved by counting possible scenarios (16 binary scenarios)

Complete the missing term(s) **(2 points)**

For <u>any</u> three events $S_1, S_1, and\ S_3$:

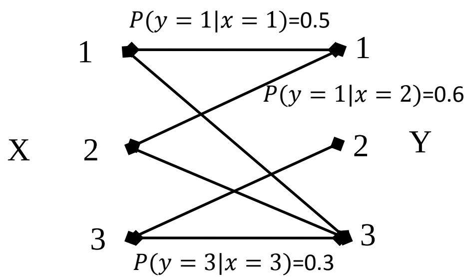
$$P(S_1 \cup S_2 \cup S_3) = P(S_1) + P(S_2) + P(S_3) - P(S_1 \cap S_2) - P(S_2 \cap S_3) - P(S_1 \cap S_3) + P(S_1 \cap S_2 \cap S_3)$$

Consider the experiment of throwing two fair dice. What is the probability that the two faces are the same given that the sum is not more than three? **(2 points)**

The only possible outcomes are (1,1), (1,2) and (2,1). The required outcome is (1,1). So the probability is 1/3.

<u>Problem 2:</u> (10 points)

Consider a system that randomly assign a variable X={1,2,3} with uniform probability, to a variable Y={1,2,3}. The conditional probability assignments are shown in the figure. Compute the following:



a) If the event y=3 was observed. What is the probability that it is coming from $= 3$. **(4 points)**

$$P(x = 3|y = 3)P(y = 3) = P(y = 3|x = 3)P(x = 3)$$

but

$$P(y = 3) = \frac{1}{3}(0.5) + \frac{1}{3}(0.4) + \frac{1}{3}(0.3) = 0.4$$

$$P(x = 3|y = 3)(0.4) = 0.3\left(\frac{1}{3}\right) = 0.25$$

b) Compute the expected value of Y. **(4 points)**

$$P(y = 3) = 0.4$$

Similarly $P(y = 1) = \frac{1}{3}(0.5 + 0.6) = 0.3667$

$$P(y = 2) = \frac{1}{3}(0.7) = 0.2333$$

Check that they sum up to 1

Expected value of $= 0.4(3) + 0.2333(2) + 0.3667(1) = 2.0333$

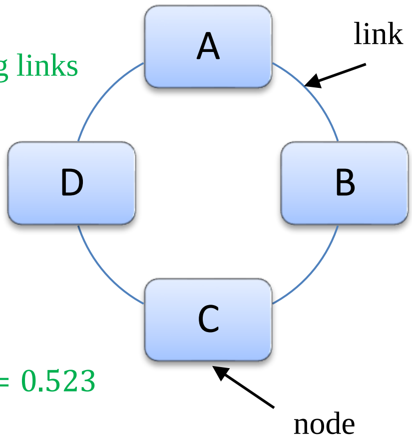
Show that If $P(A|B) > P(A)$ then $P(B|A) > P(B)$ **(2 points)**

If $P(A \mid B) = \frac{P(A \cap B)}{P(B)} > P(A)$, then $P(A \cap B) > P(A)P(B)$. Thus,

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} > \frac{P(A)P(B)}{P(A)} = P(B) \quad \text{or} \quad P(B \mid A) > P(B)$$

Problem 3:

Consider the network with nodes *A*, *B*, *C*, and *D* shown below. A network is connected if a path exists connecting a node to all other nodes. A network is disconnected when two or more links are in failure. If the links are independent and the probability of a link failure is 0,1, compute the probability of the network getting disconnected. **(5 points)**



For the network to be connected there are two scenarios.
Let X be the random variable representing the number of failing links

$$P(X=0)=\binom{4}{0}(0.1)^0(0.9)^4=0.6561$$

$$P(X=0)=\binom{4}{1}(0.1)^1(0.9)^3=0.2916$$

$$P(disconnected\ network)=1-\big(P(X=0)+P(X=1)\big)=0.523$$

The pdf of a continuous r.v. X is given by

$$f_X(x)=\begin{cases}1/3 & 0<x<1\\ 3a & 1<x<2\\ 0 & otherwise\end{cases}$$

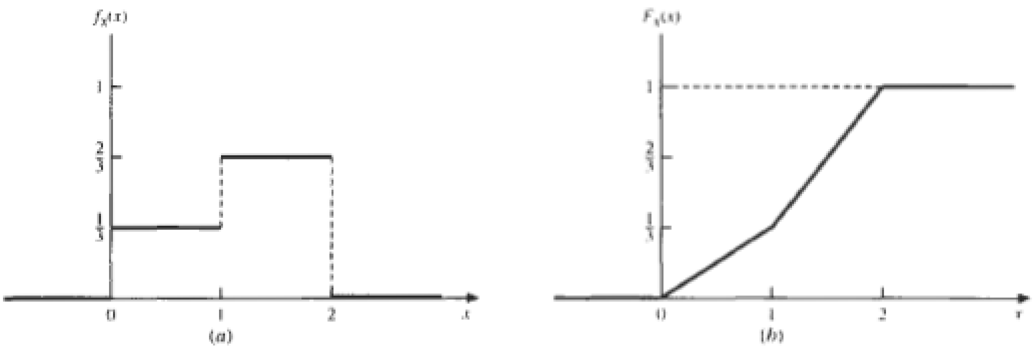
Find the value of a for a valid pdf? **(2 points)**

The total area of the pdf must be 1. By inspecting the sketch of the pdf or by integration

$$3a=2/3$$

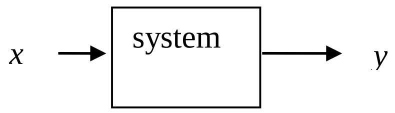
$$a=2/9$$

Sketch the CDF (show all numbers on the x-axis and the y-axis). *No need for expression only sketch* **(3 points)**



Problem 4: (10 points: 4+2+2+2)

A noise signal (Random variable X) is passed through a system with the following input output relation.



$$y = \begin{cases} -1 & -\infty < x < -1 \\ 0 & -1 \le x \le +1 \\ +1 & 1 < x < \infty \end{cases}$$

X is a Gaussian Random variable, $f_X(x) = \frac{1}{\sqrt{2\pi\sigma_X^2}} e^{-\frac{(x-a_x)^2}{2\sigma_X^2}}$, compute P($y \ge 0$), for the following cases:

a) $a_x = 0 \ and \ \sigma_X = 1$
b) $a_x = 2 \ and \ \sigma_X = 2$
c) $a_x = -1 \ and \ \sigma_X = 2$
d) $a_x = 1 \ and \ \sigma_X = 0$

$$F(-x) = 1 - F(x)$$
$$F_X(x) = F(\frac{x - a_x}{\sigma_X})$$
$$P(y \ge 0) = P(x \ge -1) = 1 - F_X(-1)$$

Case a:

$$1 - F_X(-1) = 1 - F(-1) = F(1) = \mathbf{0.8413}$$

Case b:

$$1 - F_X(-1) = 1 - F\left(\frac{-1-2}{2}\right) = 1 - F(-1.5) = F(1.5) = \mathbf{0.9332}$$

Case c: $1 - F_X(-1) = 1 - F(0) = \mathbf{0.5}$
Because we are shifting the Gaussian and taking half of the chances

Case d:
This is a deterministic event with the outcome x=1 and hence y=1 so the probability of

$$P(y \ge 0) = \mathbf{1}$$

Or comment that the Gaussian PDF is not defined for variance=0

Gaussian Table:

TABLE B-1
Values of $F(x)$ for $0 \leq x \leq 3.89$ in steps of 0.01

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9773	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.8	.9999	.9999	.9999	.9999	.9999	.9999	.9999	1.0000	1.0000	1.0000