KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

ELECTRICAL ENGINEERING DEPARTMENT

EE 315

EXAM I

DATE: Saturday 22/10/2011

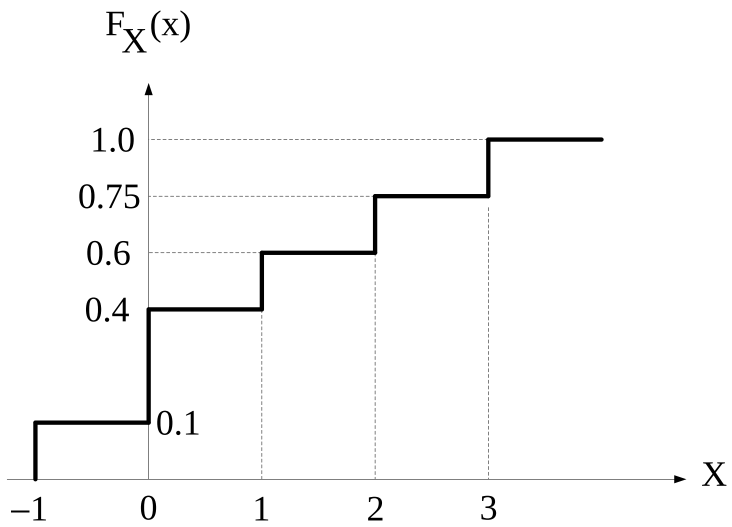
TIME: 6:00 PM-7:30 PM

ID#	
Name	**KEY**
Section#	

	Maximum Score	**Score**
Q1	**20**	
Q2	**20**	
Q3	**20**	
Q4	**20**	
TOTAL	**80**	

Problem #1 (20)

Let X be a random variable with a cumulative distribution function as shown below



(a) What is the sample space of X ? **(2.5)**

(b) Write a mathematical expression for the cumulative distribution function $F_X(x)$? **(5)**

(c) Find and plot the probability density function $f_X(x)$? **(2.5)**

(d) Find the probability $P(1 < X \le 3)$? **(5)**

(e) Find the expected value E[X] ? **(5)**

Solution

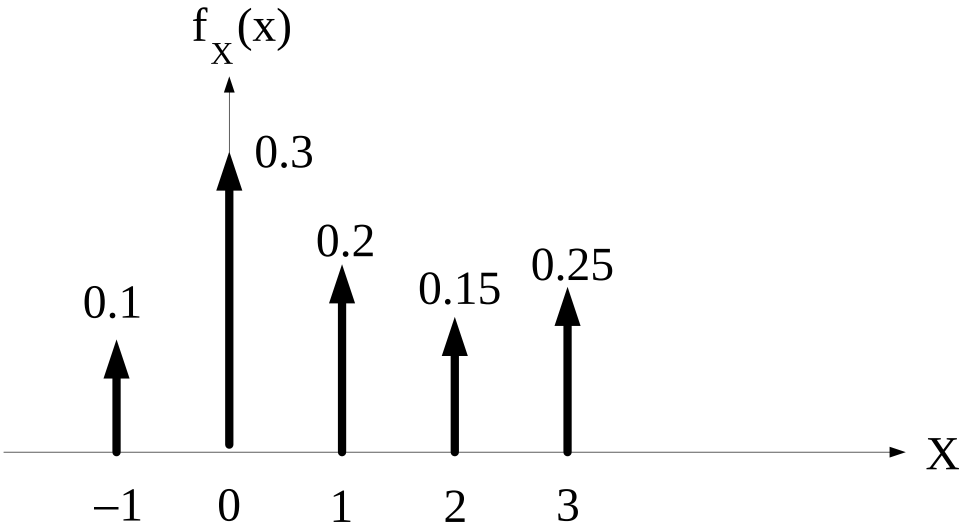
(a) $S_X = \{-1, 0, 1, 2, 3\}$

(b) $P(X=-1) = 0.1$ $\quad P(X=0) = 0.4 - 0.1 = 0.3$ $\quad P(X=1) = 0.6 - 0.2 = 0.2$

$P(X=2) = 0.75 - 0.6 = 0.15$ $\quad P(X=3) = 1 - 0.75 = 0.25$

$\Rightarrow F_X(x) = 0.1u(x+1) + 0.3u(x) + 0.2u(x-1) + 0.15u(x-2) + 0.25u(x-3)$

(c) $f_X(x) = \frac{dF_X(x)}{dx} = 0.1\delta(x+1) + 0.3\delta(x) + 0.2\delta(x-1) + 0.15\delta(x-2) + 0.25\delta(x-3)$



(d) $P(1 < X \le 3) = F_X(3) - F_X(1) = 1 - 0.6 = 0.4$

OR

$P(1 < X \le 3) = P(X = 3) + P(X = 2) = 0.25 + 0.15 = 0.4$

$$E[X] = \sum x_i P(X = x_i)$$
$$= (-1)(0.1) + (0)(0.3) + (1)(0.2) + (2)(0.15) + (3)(0.25) = 1.15$$

Problem #2 (20)

Assume that capacitors made by a manufacturer have a probability of 0.4 of being defective when new. A radio engineer purchases five capacitors for building an electronic circuit.

(a) Plot the probability density function for a random variable **"the number of defective capacitors." (12)**

(b) What is the probability that exactly one capacitor is defective of the five? **(2)**

(c) What is the probability that all five capacitors are functional? **(2)**

(d) Determine the probability that one or more capacitors are defective ? **(4)**

Solution

(a) $P(X=0) = \binom{5}{0}(0.4)^0(0.6)^5 = 0.0778$

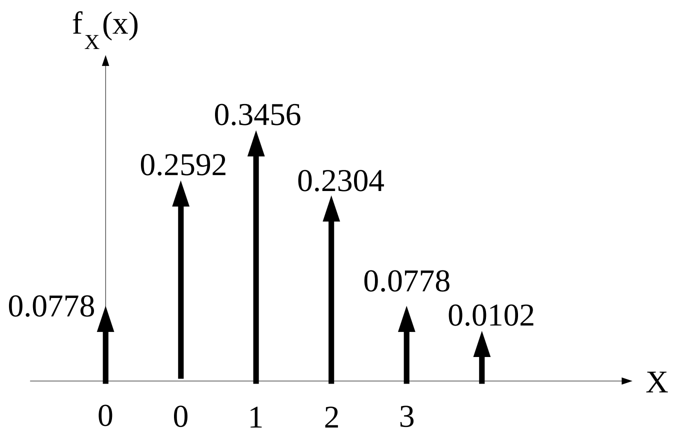
$P(X=1) = \binom{5}{1}(0.4)^1(0.6)^4 = 0.2592$

$P(X=2) = \binom{5}{2}(0.4)^2(0.6)^3 = 0.3456$

$P(X=3) = \binom{5}{3}(0.4)^3(0.6)^2 = 0.2304$

$P(X=4) = \binom{5}{4}(0.4)^4(0.6)^1 = 0.0768$

$P(X=5) = \binom{5}{5}(0.4)^5(0.6)^0 = 0.0102$



(b) $P(X=1) = 0.2592$

(c) $P(X=0) = 0.0778$

(d) $P(X \geq 1) = 1 - P(X=0) = 1 - 0.0778 = 0.9222$

OR $P(X \geq 1) = P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$

$= 0.2592 + 0.3456 + 0.2304 + 0.0768 + 0.0102 = 0.9222$

Problem #3 (20)

A box contains numbered balls with equal probability of selecting any ball . If the sample space is

S = { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}

Let the events A, B and C as follows:

A = { 0, 1, 2, 3, 4}
B = { 3, 4, 5, 6, 7}
C = { 3, 4, 5, 6, 7, 8}

(a) Find the probability P(A U B) ? **(5)**
(b) Find the probability P(A | B) ? **(5)**
(c) Are the events A and B independent , *explain* ? **(5)**
(d) Are the events A and C independent , *explain* ? **(5)**

Solution

(a) P(A U B) = P(A) + P(B) $-$ P(A B) = $\frac{5}{12} + \frac{5}{12} - \frac{2}{12} = \frac{8}{12} = \frac{2}{3}$

(b) P(A | B) = $\frac{P(A\ B)}{P(\ B)} = \frac{\frac{2}{12}}{\frac{5}{12}} = \frac{2}{5}$

(c) A and B are not independent , since P(A | B) = $\frac{2}{5} \neq$ P(A) = $\frac{5}{12}$

(d) A and C are not independent , since P(A | C) = $\frac{P(A\ C)}{P(\ C)} = \frac{\frac{2}{12}}{\frac{6}{12}} = \frac{1}{3} \neq$ P(A) = $\frac{5}{12}$

Problem #4 (20)

A random variable X has a propbabilty density function (pdf) defined by:

$$f_X(x) = \begin{cases} c\left(1-x^4\right), & -1 \le x \le 1, \\ 0, & \text{elsewhere} \end{cases}$$

(a) Find c such that $f_X(x)$ is a valid pdf ? **(4)**

(b) Find $F_X(x)$ and sketch it ? **(6)**

(c) Find $P\left[|X| < \frac{1}{2}\right]$? **(5)**

(d) Find $P[X > 0.5 | 0 < X < 1]$? **(5)**

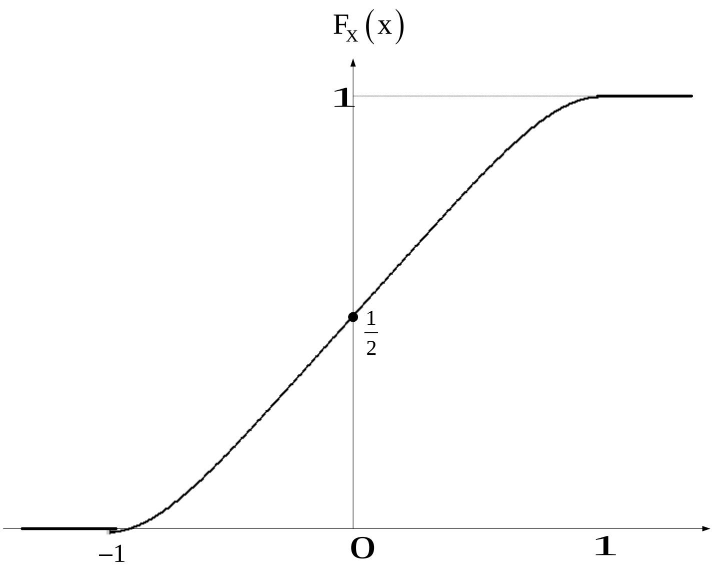
Solution

(a) $$\int_{-\infty}^{\infty} f_X(x)dx = 1 = \int_{-\infty}^{\infty} c\left(1-x^4\right)dx = c\int_{-1}^{1}\left(1-x^4\right)dx$$

$$\Rightarrow c = \frac{1}{\int_{-1}^{1}\left(1-x^4\right)dx} = \frac{1}{\left[1-\frac{x^5}{5}\right]_{-1}^{1}} = \frac{1}{\frac{8}{5}} = \frac{5}{8}$$

(b) $F_X(x) = 0 \quad X < -1$

$$F_X(x) = \int_{-1}^{x} f_X(\varepsilon)d\varepsilon = \frac{5}{8}\int_{-1}^{x}\left(1-\varepsilon^4\right)d\varepsilon = \frac{5}{8}x - \frac{1}{8}x^5 + \frac{1}{2}$$



(c) $P\left(|X| < \frac{1}{2}\right) = P\left(-\frac{1}{2} < X < \frac{1}{2}\right) = \frac{5}{8}\int_{-\frac{1}{2}}^{\frac{1}{2}}\left(1 - x^4\right)dx = \frac{79}{128} = 0.6172$

(d) $P\left(X > 0.5 \mid 0 < X < 1\right) = \frac{P\left(X > 0.5 \cap 0 < X < 1\right)}{P\left(0 < X < 1\right)} = \frac{P\left(0.5 < X < 1\right)}{P\left(0 < X < 1\right)}$

$$= \frac{\frac{5}{8}\int_{\frac{1}{2}}^{1}\left(1 - x^4\right)dx}{\frac{5}{8}\int_{0}^{1}\left(1 - x^4\right)dx} = \frac{\frac{49}{256}}{\frac{1}{2}} = \frac{49}{128}$$