King Fahd University of Petroleum and Minerals

Department of Electrical Engineering
EE 315 Probabilistic Methods in Electrical Engineering

Major Exam I
Saturday, 14 Nov. 2009
6:00 pm – 8:00pm

Name: ________________________________

ID: __________________________________

Sections: **1**

Instructor: Samir Alghadhban

Problem	Score	Out of
1		10
2		10
3		10
4		10
Total		40

Good luck!

Problem 1

The parts of this problem are unrelated to each other

a) *Members of the KFUPM football team win each match they play with probability of 0.85. If the results of matches they play are independent of each other, find the probability that they will win 9 of their next 13 matches.*

$$P(\text{winning any 9 of 13 matches}) = \binom{13}{9}(0.85)^9(0.15)^4$$
$$= 0.08384$$

b) *State if the function* $F_X(x) = \frac{x}{8}\left[u(x-4) - u(x-8)\right]$ *is a valid CDF of the random variable* X *. If it is not, state the property of CDFs that is violated.*

The function $F_X(x) = \frac{x}{8}\left[u(x-4) - u(x-8)\right]$ is not a CDF because of any of two reasons

1) $F_X(\infty) = 0$ while a valid CDF should be equal to 1 at ∞.

2) $F_X(7) = \frac{7}{8}$ and $F_X(9) = 0$, while a valid CDF must increase or remain constant as x increases.

c) *In an experiment, two events* A *and* B *have probabilities* $P(A) = 0.25$, $P(B) = 0.40$, *and* $P(A \cup B) = 0.65$. *Determine if these two events are (show your work and explain why?)*

d)

Since

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$
$$= 0.25 + 0.40 - 0.65$$
$$= 0$$

➔ A and B are mutually exclusive

Mutually exclusive events CANNOT be independent

i. *Mutually exclusive* *(Yes or No):* YES

ii. *Independent* *(Yes or No):* NO

d) *A bag contains 6 balls numbered from 1 to 6, if you pick one ball at a time and hold it, how many ordered sequences can you get if you pick a total of 4 balls.*

This is a permutations problem (Picking without replacement and ordering the picked items)

$$P_4^6 = \frac{6!}{(6-4)!} = 360$$

e) *Consider a random variable with exponential pdf, find its characteristic function. {hint: the exponential pdf is $f_x(x) = \frac{1}{b} e^{\frac{-(x-a)}{b}}$ for $x > a$. }*

$$\Phi_X(\omega) = \int_a^\infty \frac{1}{b} e^{-(x-a)/b} e^{j\omega x} dx$$

$$= \frac{e^{a/b}}{b} \int_a^\infty e^{-(1/b - j\omega)x} dx$$

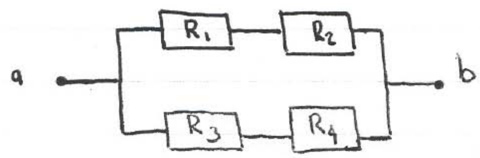
$$= \frac{e^{a/b}}{b} \left[\frac{e^{-(1/b - j\omega)x}}{-(1/b - j\omega)} \right]_a^\infty$$

$$= \frac{e^{j\omega a}}{1 - j\omega b}$$

Problem 2

2.1 In a communication system, the signal sent from point a to b arrives by two paths in parallel. Over each path, the signal passes through two repeaters in series. In the first path, each repeater has a probability of failing of 0.005. In the second path, the probability of failing for each repeater is 0.008. All repeaters fail independently form each other.

a) Draw a sketch of the problem.
b) Find the probability that the signal will not arrive at point b.



Let $R_i = \{\text{Relay } R_i \text{ fails}\}$ $(i = 1, 2, 3, 4)$ be the event for the R_i relay to fail.

$$P(R_1) = P(R_2) = 0.005$$
$$P(R_3) = P(R_4) = 0.008$$

The probability that the signal will not arrive at point b is:

$$P[(R_1 \text{ opens or } R_2 \text{ opens}) \text{ and } (R_3 \text{ opens or } R_4 \text{ opens})]$$

$$= P[(R_1 \cup R_2) \cap (R_3 \cup R_4)]$$

$$= P(R_1 \cup R_2)\, P(R_3 \cup R_4) \qquad [\text{independent events}].$$

$$= [P(R_1) + P(R_2) - P(R_1 \cap R_2)][P(R_3) + P(R_4) - P(R_3 \cap R_4)]$$

$$= [P(R_1) + P(R_2) - P(R_1)P(R_2)][P(R_3) + P(R_4) - P(R_3)P(R_4)]$$

$$= 0.00016$$

2.2 The Characteristic function of the Laplace density is known to be

$$\Phi_X(w) = \frac{e^{jm\omega}}{1+(b\omega)^2}$$

Use this result to find the **mean**, **second moment**, and **the variance** of the random variable ***X***.

$$m_1 = E[X]$$

$$= -j \left. \frac{d\Phi_X(w)}{dw} \right|_{w=0}$$

$$= m$$

and

$$m_2 = (-j)^2 \left. \frac{d^2\Phi_X(w)}{dw^2} \right|_{w=0}$$

$$= 2b^2 + m^2$$

$$\therefore \sigma_X^2 = m_2 - m_1^2$$

$$= 2b^2$$

Problem 3

An experiment is performed by tossing a fair die **two times** (each face of the die has equal probability of being at top). However, the 6 faces of the die are numbered such that

1 face is numbered 1
2 faces are numbered 2
3 faces are numbered 3

The numbers appearing on top are recorded **in order**.

A random variable X maps each output in the sample space of the experiment to the point on the real line that is equal to the sum of the two numbers from the tosses.

a) Show the sample space of the experiment and the mapping of X from the sample space to the real line,
b) Sketch the pdf of X (show all important values on your sketch),
c) Sketch the CDF of X (show all important values on your sketch),
d) Find $P\{X \leq 2.5\}$
e) Find $P\{1.8 < X \leq 4.8\}$
f) Find $P\{X > 3.3\}$
g) Find $F_X(3)$

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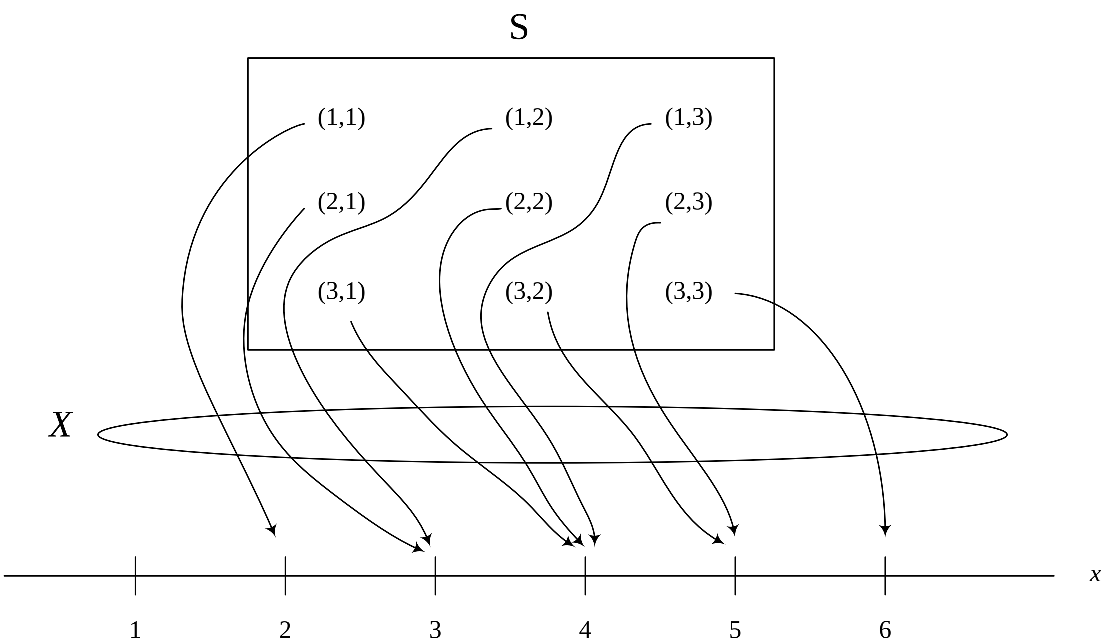
a)



b) We know that there is face with 1, 2 faces with 2, and three faces with 3, and that the throws are independent, so

P(a die shows 1) = 1/6
P(a die shows 2) = 2/6
P(a die shows 3) = 3/6

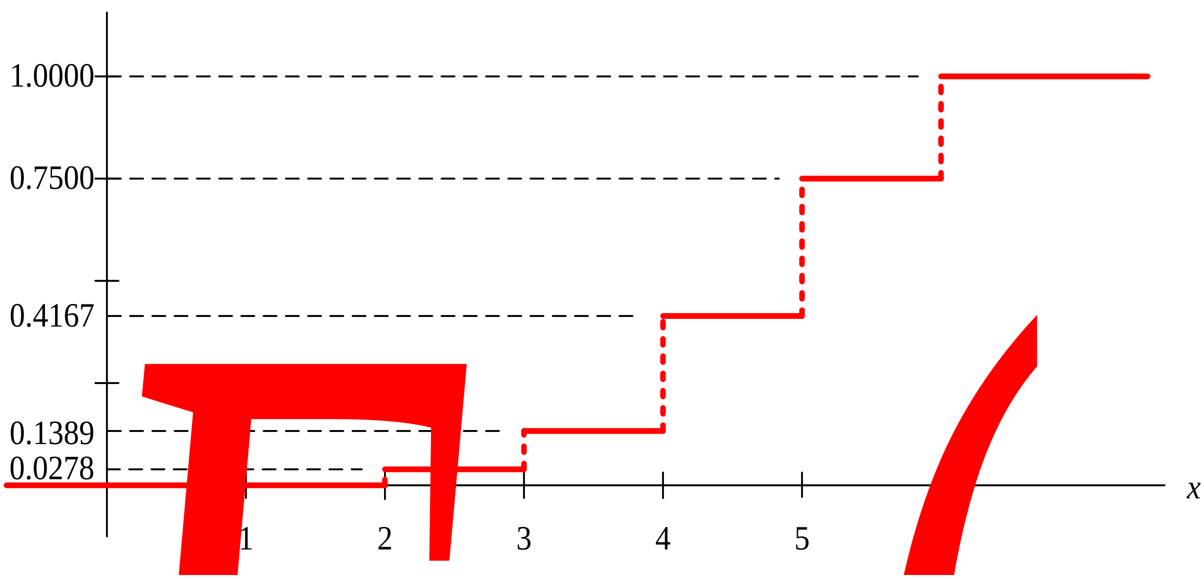
and

P{s = (1,1)} = (1/6)(1/6) = 1/36
P{s = (1,2)} = (1/6)(2/6) = 2/36
P{s = (1,3)} = (1/6)(3/6) = 3/36
P{s = (2,1)} = (2/6)(1/6) = 2/36
P{s = (2,2)} = (2/6)(2/6) = 4/36
P{s = (2,3)} = (2/6)(3/6) = 6/36
P{s = (3,1)} = (3/6)(1/6) = 3/36
P{s = (3,2)} = (3/6)(2/6) = 6/36
P{s = (3,3)} = (3/6)(3/6) = 9/36

So,

P(X = 2) = 1/36
P(X = 3) = 2/36 + 2/36 = 4/36
P(X = 4) = 3/36 + 4/36 + 3/36 = 10/36
P(X = 5) = 6/36 + 6/36 = 12/36
P(X = 6) = 9/36

c) Now, the CDF becomes



d) $P\{X \le 2.5\} = F_X(2.5) = 0.0278$

e) $P\{1.8 < X \le 4.8\} = F_X(4.5) - F_X(1.8) = 0.4167 - 0 = 0.4167$

f) $P\{X > 3.3\} = 1 - P\{X \le 3.3\} = 1 - F_X(3.3) = 1 - 0.1389 = 0.8611$

f) Since $F_X(x^+) = F_X(x)$ for any CDF, then $F_X(3) = F_X(3^+) = 0.1389$

Problem 4

Let X be a random variable with a Gaussian probability density function given as

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma_x^2}} e^{\frac{-(x - a_x)^2}{2\sigma_x^2}}$$

If $a_X = 1$ and $\sigma_X^2 = 4$ and let the events A and B be defined as

A = { 0 < X < 2 }
B = { –1 < X < 1 }

(a) P (| X | < 3) ?
(b) P(A $\cup$ B) ?
(c) Are events A and B statistically independent ? Explain why?
(d) Let $Y = cX^2$, where $c > 0$, find the pdf of Y. { Show your work and all steps}

a)

$$\begin{aligned} P(|x| < 3) &= P(-3 < x < 3) = F_x(3) - F_x(-3) \\ &= F\left(\frac{3-1}{2}\right) - F\left(\frac{-3-1}{2}\right) \\ &= F(1) - F(-2) = F(1) - 1 + F(2) \\ &= 0.8413 - 1 + 0.9773 \\ &= 0.8186 \end{aligned}$$

b)

$$\begin{aligned} P(A \cup B) &= P(-1 < x < 2) = F\left(\frac{2-1}{2}\right) - F\left(\frac{-1-1}{2}\right) \\ &= F(0.5) - F(-1) = F(0.5) - 1 + F(1) \\ &= 0.5328 \end{aligned}$$

c)

Statistically independent if $P(A \cap B) = P(A)P(B)$

$$P(A \cap B) = P(0 < x < 1) = F(0) - F(-0.5) = 0.1915$$

$$P(A) = P(0 < x < 2) = 2F(0.5) - 1 = 0.383$$

$$P(B) = P(-1 < x < 1) = F(0) - F(-1) = 0.3413$$

Since $P(A \cap B) \neq P(A)P(B)$ ➔ they are not independent.

d) see textbook example 3.4-2 on page 91.