

King Fahd University of Petroleum & Minerals
Electrical Engineering Department
EE315: Probabilistic Methods in Electrical Engineering (112)
Quiz 6: Operation on Multiple Random Variable- Expectation

- 1 points for not writing your serial number

Name: KEY

Two random variables X and Y have means $\bar{X} = 1$ and $\bar{Y} = 2$, variances $\sigma_X^2 = 4$, and $\sigma_Y^2 = 1$, and the correlation coefficient $\rho_{XY} = 0.4$. New random variables V and W are defined by

$$V = -X + 2Y \quad \text{and} \quad W = X + 3Y$$

For the new variables V and W , find:

- The means. (2 points)
- The variances. (4 points)
- The correlation. (2 points)
- The correlation coefficient, ρ_{VW} . (2 points)

Here $\bar{X} = 1, \bar{Y} = 2, \sigma_X^2 = 4, \sigma_Y^2 = 1, \rho_{XY} = 0.4$.

(a) $\bar{V} = E[V] = E[-X + 2Y] = -\bar{X} + 2\bar{Y} = -1 + 4 = 3$.

$\bar{W} = E[W] = E[X + 3Y] = \bar{X} + 3\bar{Y} = 1 + 6 = 7$.

(b) $\sigma_V^2 = E[(V - \bar{V})^2] = E[(-X + 2Y + \bar{X} - 2\bar{Y})^2]$
 $= E[\{- (X - \bar{X}) + 2(Y - \bar{Y})\}^2] = \overline{(X - \bar{X})^2} - 4 \overline{(X - \bar{X})(Y - \bar{Y})} + 4 \overline{(Y - \bar{Y})^2}$
 $= \sigma_X^2 - 4 C_{XY} + 4 \sigma_Y^2$. But $\rho_{XY} = \frac{C_{XY}}{\sigma_X \sigma_Y}$
 $= 0.4$, so $C_{XY} = 0.4 \sigma_X \sigma_Y = 0.8$ and
 $\sigma_V^2 = 4 - 4(0.8) + 4 = 4.8$. Next, $\sigma_W^2 = E[(W - \bar{W})^2]$
 $= E[\{(X - \bar{X}) + 3(Y - \bar{Y})\}^2] = \sigma_X^2 + 6 C_{XY} + 9 \sigma_Y^2 = 17.8$.

(c) $R_{VW} = E[VW] = E[(-X + 2Y)(X + 3Y)] = -\bar{X}^2 - \bar{X}\bar{Y} + 6\bar{Y}^2$
 $= -[\sigma_X^2 + \bar{X}^2] - R_{XY} + 6[\sigma_Y^2 + \bar{Y}^2] = -[4 + 1] - [C_{XY} + \bar{X}\bar{Y}] + 6[1 + 4]$
 $= -5 - [0.8 + 2] + 6(5) = 22.2$.

(d) $\rho_{VW} = C_{VW} / \sigma_V \sigma_W = (R_{VW} - \bar{V}\bar{W}) / \sigma_V \sigma_W$
 $= (22.2 - 21) / \sqrt{4.8(17.8)} \approx 0.1298$.