Trigonometric Identities:

$$\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$$
$$\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$$
$$\cos\left(x \pm \frac{\pi}{2}\right) = \mp \sin(x)$$
$$\sin\left(x \pm \frac{\pi}{2}\right) = \pm \cos(x)$$
$$\cos(2x) = \cos^2(x) - \sin^2(x)$$
$$\sin(2x) = 2\sin(x)\cos(x)$$
$$2\cos(x) = e^{jx} + e^{-jx}$$
$$2j\sin(x) = e^{jx} - e^{-jx}$$
$$2\cos(x)\cos(y) = \cos(x-y) + \cos(x+y)$$
$$2\sin(x)\sin(y) = \cos(x-y) - \cos(x+y)$$
$$2\sin(x)\cos(y) = \sin(x-y) + \sin(x+y)$$
$$2\cos^2(x) = 1 + \cos(2x)$$
$$2\sin^2(x) = 1 - \cos(2x)$$
$$4\cos^3(x) = 3\cos(x) + \cos(3x)$$
$$4\sin^3(x) = 3\sin(x) - \sin(3x)$$
$$8\cos^4(x) = 3 + 4\cos(2x) + \cos(4x)$$

Fourier Transform Pairs

Pair	$x(t)$	$X(\omega)$
1	$\alpha\delta(t)$	α
2	$\alpha/2\pi$	$\alpha\delta(\omega)$
3	$u(t)$	$\pi\delta(\omega) + (1/j\omega)$
4	$\frac{1}{2}\delta(t) - \frac{1}{j2\pi t}$	$u(\omega)$
5	$\text{rect}(t/\tau)$	$\tau\,\text{Sa}(\omega\tau/2)$
6	$(W/\pi)\text{Sa}(Wt)$	$\text{rect}\,(\omega/2W)$
7	$\text{tri}\,(t/\tau)$	$\tau\,\text{Sa}^2(\omega\tau/2)$
8	$(W/\pi)\text{Sa}^2(Wt)$	$\text{tri}\,(\omega/2W)$
9	$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$
10	$\delta(t-\tau)$	$e^{-j\omega\tau}$
11	$\cos(\omega_0 t)$	$\pi[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)]$
12	$\sin(\omega_0 t)$	$-j\pi[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)]$
13	$u(t)\cos(\omega_0 t)$	$\frac{\pi}{2}[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)] + \frac{j\omega}{\omega_0^2 - \omega^2}$
14	$u(t)\sin(\omega_0 t)$	$-j\frac{\pi}{2}[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)] + \frac{\omega_0}{\omega_0^2 - \omega^2}$
15	$u(t)e^{-\alpha t}$	$\frac{1}{\alpha + j\omega}$
16	$u(t)te^{-\alpha t}$	$\frac{1}{(\alpha + j\omega)^2}$
17	$u(t)t^2e^{-\alpha t}$	$\frac{2}{(\alpha + j\omega)^3}$
18	$u(t)t^3e^{-\alpha t}$	$\frac{6}{(\alpha + j\omega)^4}$
19	$e^{-\alpha\|t\|}$	$\frac{2\alpha}{\alpha^2 + \omega^2}$
20	$e^{-t^2/(2\sigma^2)}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$

In Table E-1 of Fourier transform pairs, we define

$$u(\xi) = \begin{cases} 1 & \xi > 0 \\ 0 & \xi < 0 \end{cases}$$

$$\text{rect}\,(\xi) = \begin{cases} 1 & |\xi| < \frac{1}{2} \\ 0 & |\xi| > \frac{1}{2} \end{cases}$$

$$\text{Sa}(\xi) = \frac{\sin(\xi)}{\xi}$$

$$\text{tri}\,(\xi) = \begin{cases} 1 - |\xi| & |\xi| < 1 \\ 0 & |\xi| > 1 \end{cases}$$

$$x(t) \leftrightarrow X(\omega)$$

and let α, τ, σ, ω_0, and W be real constants.

Gaussian Table:

TABLE B-1
Values of $F(x)$ for $0 \le x \le 3.89$ in steps of 0.01

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9773	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.8	.9999	.9999	.9999	.9999	.9999	.9999	.9999	1.0000	1.0000	1.0000

Exponential Functions

$$\int e^{ax}\,dx = \frac{e^{ax}}{a} \qquad a \text{ real or complex}$$

$$\int xe^{ax}\,dx = e^{ax}\left[\frac{x}{a} - \frac{1}{a^2}\right] \qquad a \text{ real or complex}$$

$$\int x^2e^{ax}\,dx = e^{ax}\left[\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right] \qquad a \text{ real or complex}$$

$$\int x^3e^{ax}\,dx = e^{ax}\left[\frac{x^3}{a} - \frac{3x^2}{a^2} + \frac{6x}{a^3} - \frac{6}{a^4}\right] \qquad a \text{ real or complex}$$

$$\int e^{ax}\sin(x)\,dx = \frac{e^{ax}}{a^2+1}[a\sin(x) - \cos(x)]$$

$$\int e^{ax}\cos(x)\,dx = \frac{e^{ax}}{a^2+1}[a\cos(x) + \sin(x)]$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$
$$P(A) = \sum_{n=1}^{N} P(A \mid B_n) P(B_n)$$
$$P(B_n \mid A) = \frac{P(A \mid B_n) P(B_n)}{P(A \mid B_1) P(B_1) + \dots + P(A \mid B_n) P(B_n)}$$
$$\binom{N}{k} = \frac{N!}{k!(N-k)!}$$
$$P(A \ \text{occurs} \ k \ \text{times}) = \binom{N}{k} p^k (1-p)^{N-k}$$

$$F_X(x) = P\{X \le x\}$$
$$F_X(x) = \sum_{i=0}^{N} P(x_i) u(x - x_i)$$
$$f_X(x) = \frac{dF_X(x)}{dx}$$
$$F_X(x) = \int_{-\infty}^{x} f_X(\xi) d\xi$$
$$F_X(x) = F\left(\frac{x - a_X}{\sigma_X}\right)$$
$$F_X(x \mid B) = P\{X \le x \mid B\} = \frac{P\{X \le x \cap B\}}{P(B)}$$
$$f_X(x \mid B) = \frac{dF_X(x \mid B)}{dx}$$

$$E[X] = \overline{X} = \sum_{i=1}^{N} x_i P(x_i)$$
$$E[X] = \int_{-\infty}^{\infty} x f_X(x) dx$$
$$E[g(x)] = \int_{-\infty}^{\infty} g(x) f_X(x) dx$$
$$m_n = E[X^n]$$
$$\mu_n = E[(X - \overline{X})^n]$$
$$F_X(x) = F\left(\frac{x - a_X}{\sigma_X}\right)$$
$$\Phi_X(\omega) = E\left[e^{j\omega x}\right]$$
$$\Phi_X(\omega) = \int_{-\infty}^{\infty} f_X(x) e^{j\omega x} dx$$
$$m_n = (-j)^n \left. \frac{d^n \Phi_X(\omega)}{d\omega^n} \right|_{\omega=0}$$
$$M_X(v) = \int_{-\infty}^{\infty} f_X(x) e^{vx} dx$$
$$m_n = \left. \frac{d^n M_X(v)}{dv^n} \right|_{v=0}$$
$$f_Y(y) = \sum_n \frac{f_X(x_n)}{\left| \left. \frac{dy}{dx} \right|_{x=x_n} \right|}$$

$$F_{X,Y}(x, y) = P\{X \le x, Y \le y\}$$
$$P\{x1 < X \le x2,\ y1 < Y \le y2\} =$$
$$F_{X,Y}(x2, y2) + F_{X,Y}(x1, y2) - F_{X,Y}(x1, y2) - F_{X,Y}(x2, y1)$$
$$F_{X,Y}(x, \infty) = F_X(x);$$
$$f_{X,Y}(x, y) = \frac{\partial^2 F_{X,Y}(x, y)}{\partial x \, \partial y}$$
$$P\{x1 < X \le x2,\ y1 < Y \le y2\} = \int_{y1}^{y2} \int_{x1}^{x2} f_{X,Y}(x, y) dx dy$$
$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x, y) dy; \ f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x, y) dx$$
$$f_X(x) = \frac{dF_X(x)}{dx}; \ f_Y(y) = \frac{dF_Y(y)}{dy}$$
$$f_X(x \mid y) = \frac{f_{X,Y}(x, y)}{f_Y(y)};$$

Gaussian pdf: $f_X(x) = \frac{1}{\sqrt{2\pi\sigma_x^2}} e^{\frac{-(x - a_x)^2}{2\sigma_x^2}}$

Poisson pdf: $f_X(x) = e^{-b} \sum_{k=0}^{\infty} \frac{b^k}{k!} u(x - k)$ where $b = \lambda T$

$$\overline{g} = E[g(x)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x, y) f_{X,Y}(x, y) dx dy$$
$$m_{nk} = E[X^n Y^k] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^n y^k f_{X,Y}(x, y) dx dy$$
$$R_{XY} = m_{11} = E[XY]$$
$$\mu_{nk} = E[(X - \overline{X})^n (Y - \overline{Y})^k] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (X - \overline{X})^n (Y - \overline{Y})^k f_{X,Y}(x, y) dx dy$$
$$C_{XY} = \mu_{11} = E[(X - \overline{X})(Y - \overline{Y})] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (X - \overline{X})(Y - \overline{Y}) f_{X,Y}(x, y) dx dy$$
$$C_{XY} = R_{XY} - \overline{X}\,\overline{Y} = R_{XY} - E[X]E[Y]$$
$$\rho = \frac{C_{XY}}{\sigma_X \sigma_Y}; \ -1 \le \rho \le 1$$
$$f_{u,v}(u, v) = f_{X,Y}(x, y) |J|; \qquad \text{where} \ J = \det \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{bmatrix}$$

$$\mu_X(t) = E[X(t)] = \int_{-\infty}^{\infty} x f_{X(t)}(x) dx$$
$$R_X(t_1, t_2) = E[X(t_1) X(t_2)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x_1 x_2 f_{X(t_1), X(t_2)}(x_1 x_2) dx_1 dx_2$$
$$< X(t) > = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} X(t) dt; \ \mu_Y(t) = E[Y(t)] = \mu_X H(0)$$
$$h(t) = \int_{-\infty}^{\infty} H(f) \exp(j 2\pi f t) df; \quad H(f) = \int_{-\infty}^{\infty} h(t) \exp(-j 2\pi f t) dt$$
$$S_{XX}(f) = \int_{-\infty}^{\infty} R_{XX}(\tau) \exp(-j 2\pi f \tau) d\tau \Leftrightarrow R_{XX}(\tau) = \int_{-\infty}^{\infty} S_{XX}(f) \exp(j 2\pi f \tau) df$$
$$S_{YY}(f) = |H(f)|^2 S_{XX}(f)$$
$$R_{YY}(\tau) = R_{XX}(\tau) * h(-\tau) * h(\tau)$$