## King Fahd University of Petroleum & Minerals

Electrical Engineering Department EE207: Signals and Systems (122)

**Quiz 6: Laplace Transform** 

- 1 points for not writing your serial number

Name: Sec.

Find the Laplace transform for the following signal

$$x(t) = \begin{cases} 1 & 0 \le t \le 1 \\ 0 & otherwise \end{cases}$$

$$x(t) = u(t) - u(t-1)$$

$$X(s) = \frac{1}{s} - \frac{e^{-s}}{s} = \frac{1 - e^{-s}}{s}$$

Find the inverse Laplace transform of  $\frac{2s^2-8}{(s^2+4)^2}$ 

$$\frac{2(s^2-4)}{(s^2+4)^2}$$

Last pair in the table scaled by 2

Find the Laplace transform of  $x(t) = t^2 u(t)$ 

$$\frac{2}{c^3}$$

Find the Laplace transform of  $x(t) = e^{-t}t^2u(t)$ 

$$\frac{2}{(s+1)^3}$$

$f(t), t \ge 0$	F(s)	ROC
$1. \delta(t)$	1	All s
2. <i>u</i> ( <i>t</i> )	$\frac{1}{s}$	Re(s) > 0
3. <i>t</i>	$\frac{1}{s^2}$	Re(s) > 0
4. <i>t</i> <sup>n</sup>	$\frac{n!}{s^{n+1}}$	Re(s) > 0
5. $e^{-at}$	$\frac{1}{s+a}$	Re(s) > -a
6. $te^{-at}$	$\frac{1}{(s+a)^2}$	Re(s) > -a
7. $t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$	Re(s) > -a
8. sin <i>bt</i>	$\frac{b}{s^2+b^2}$	Re(s) > 0
9. cos <i>bt</i>	$\frac{s}{s^2+b^2}$	Re(s) > 0
10. $e^{-at} \sin bt$	$\frac{b}{(s+a)^2+b^2}$	Re(s) > -a
11. $e^{-at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$	Re(s) > -a
12. <i>t</i> sin <i>bt</i>	$\frac{2bs}{(s^2+b^2)^2}$	Re(s) > 0
13. <i>t</i> cos <i>bt</i>	$\frac{s^2 - b^2}{(s^2 + b^2)^2}$	Re(s) > 0

Name	Property
1. Linearity, (7.10)	$\mathcal{L}[a_1f_1(t) + a_2f_2(t)] = a_1F_1(s) + a_2F_2(s)$
2. Derivative, (7.15)	$\mathscr{L}\left[\frac{df(t)}{dt}\right] = sF(s) - f(0^+)$
3. <i>n</i> th-order derivative, (7.29)	$\mathcal{L}\left[\frac{d^n f(t)}{dt^n}\right] = s^n F(s) - s^{n-1} f(0^+)$ $- \dots - s f^{(n-2)}(0^+) - f^{(n-1)}(0^+)$
4. Integral, (7.31)	$\mathscr{L}\left[\int_0^t f(\tau)d\tau\right] = \frac{F(s)}{s}$
5. Real shifting, (7.22)	$\mathcal{L}[f(t-t_0)u(t-t_0)] = e^{-t_0s}F(s)$
6. Complex shifting, (7.20)	$\mathcal{L}[e^{-at}f(t)] = F(s+a)$
7. Initial value, (7.36)	$\lim_{t \to 0^+} f(t) = \lim_{s \to \infty} sF(s)$
8. Final value, (7.39)	$\lim_{t \to \infty} f(t) = \lim_{s \to 0} s F(s)$
9. Multiplication by $t$ , (7.34)	$\mathcal{L}[tf(t)] = -\frac{dF(s)}{ds}$
10. Time transformation, (7.42) $(a > 0; b \ge 0)$	$\mathcal{L}[f(at-b)u(at-b)] = \frac{e^{-sb/a}}{a}F\left(\frac{s}{a}\right)$
11. Convolution	$\mathcal{L}^{-1}[F_1(s)F_2(s)] = \int_0^t f_1(t-\tau)f_2(\tau) d\tau$
	$= \int_0^t f_1(\tau) f_2(t-\tau) d\tau$
12. Time periodicity	$\mathcal{L}[f(t)] = \frac{1}{1 - e^{-sT}} F_1(s)$ , where
$[f(t) = f(t+T)], t \ge 0$	$F_1(s) = \int_0^T f(t)e^{-st} dt$