## Serial \#

## King Fahd University of Petroleum \& Minerals

Electrical Engineering Department
EE207: Signals and Systems (122)

- 1 points for not writing your serial
Quiz 6: Laplace Transform
Name:

Find the Laplace transform for the following signal

$$
\begin{gathered}
x(t)=\left\{\begin{array}{cc}
1 & 0 \leq t \leq 1 \\
0 & \text { otherwise }
\end{array}\right. \\
x(t)=u(t)-u(t-1) \\
X(s)=\frac{1}{s}-\frac{e^{-s}}{s}=\frac{1-e^{-s}}{s}
\end{gathered}
$$

Find the inverse Laplace transform of $\frac{2 s^{2}-8}{\left(s^{2}+4\right)^{2}}$

$$
\frac{2\left(s^{2}-4\right)}{\left(s^{2}+4\right)^{2}}
$$

Last pair in the table scaled by 2

$$
2 t \cos (2 t) u(t)
$$

Find the Laplace transform of $x(t)=t^{2} u(t)$

$$
\frac{2}{s^{3}}
$$

Find the Laplace transform of $x(t)=e^{-t} t^{2} u(t)$

$$
\frac{2}{(s+1)^{3}}
$$

| $\mathrm{f}(\mathrm{t}), \mathrm{t} \geqq 0$ | F(s) | ROC |
| :---: | :---: | :---: |
| 1. $\delta(t)$ | 1 | All s |
| 2. $u(t)$ | $\frac{1}{s}$ | $\operatorname{Re}(s)>0$ |
| 3. $t$ | $\frac{1}{s^{2}}$ | $\operatorname{Re}(s)>0$ |
| 4. $t^{n}$ | $\frac{n!}{s^{n+1}}$ | $\operatorname{Re}(s)>0$ |
| 5. $e^{-a t}$ | $\frac{1}{s+a}$ | $\operatorname{Re}(s)>-a$ |
| 6. $t e^{-a t}$ | $\frac{1}{(s+a)^{2}}$ | $\operatorname{Re}(s)>-a$ |
| 7. $t^{n} e^{-a t}$ | $\frac{n!}{(s+a)^{n+1}}$ | $\operatorname{Re}(s)>-a$ |
| 8. $\sin b t$ | $\frac{b}{s^{2}+b^{2}}$ | $\operatorname{Re}(s)>0$ |
| 9. $\cos b t$ | $\frac{s}{s^{2}+b^{2}}$ | $\operatorname{Re}(s)>0$ |
| 10. $e^{-a t} \sin b t$ | $\frac{b}{(s+a)^{2}+b^{2}}$ | $\operatorname{Re}(s)>-a$ |
| 11. $e^{-a t} \cos b t$ | $\frac{s+a}{(s+a)^{2}+b^{2}}$ | $\operatorname{Re}(s)>-a$ |
| 12. $t \sin b t$ | $\frac{2 b s}{\left(s^{2}+b^{2}\right)^{2}}$ | $\operatorname{Re}(s)>0$ |
| 13. $t \cos b t$ | $\frac{s^{2}-b^{2}}{\left(s^{2}+b^{2}\right)^{2}}$ | $\operatorname{Re}(s)>0$ |
| Name |  | Property |
| 1. Linearity, (7.10) |  | $\mathscr{L}\left[a_{1} f_{1}(t)+a_{2} f_{2}(t)\right]=a_{1} F_{1}(s)+a_{2} F_{2}(s)$ |
| 2. Derivative, (7.15) |  | $\mathscr{L}\left[\frac{d f(t)}{d t}\right]=s F(s)-f\left(0^{+}\right)$ |
| 3. $n$ th-order derivative, (7.29) |  | $\begin{aligned} & \mathscr{L}\left[\frac{d^{n} f(t)}{d t^{n}}\right]=s^{n} F(s)-s^{n-1} f\left(0^{+}\right) \\ & -\cdots-s f^{(n-2)}\left(0^{+}\right)-f^{(n-1)}\left(0^{+}\right) \end{aligned}$ |
| 4. Integral, (7.31) |  | $\mathscr{L}\left[\int_{0}^{t} f(\tau) d \tau\right]=\frac{F(s)}{s}$ |
| 5. Real shifting, (7.22) |  | $\mathscr{L}\left[f\left(t-t_{0}\right) u\left(t-t_{0}\right)\right]=e^{-t_{0} s} F(s)$ |
| 6. Complex shifting, (7.20) |  | $\mathscr{L}\left[e^{-a t} f(t)\right]=F(s+a)$ |
| 7. Initial value, (7.36) |  | $\lim _{t \rightarrow 0^{+}} f(t)=\lim _{s \rightarrow \infty} s F(s)$ |
| 8. Final value, (7.39) |  | $\lim _{t \rightarrow \infty} f(t)=\lim _{s \rightarrow 0} s F(s)$ |
| 9. Multiplication by $t$, (7.34) |  | $\mathscr{L}[t f(t)]=-\frac{d F(s)}{d s}$ |
| 10. Time transformation, (7.42) $(a>0 ; b \geqq 0)$ |  | $\mathscr{L}[f(a t-b) u(a t-b)]=\frac{e^{-s b / a}}{a} F\left(\frac{s}{a}\right)$ |
| 11. Convolution |  | $\mathscr{L}^{-1}\left[F_{1}(s) F_{2}(s)\right]=\int_{0}^{t} f_{1}(t-\tau) f_{2}(\tau) d \tau$ |
|  |  | $=\int_{0}^{t} f_{1}(\tau) f_{2}(t-\tau) d \tau$ |
| 12. Time periodicity |  | $\mathscr{L}[f(t)]=\frac{1}{1-e^{-s T}} F_{1}(s)$, where |
| $[f(t)=f(t+T)], t \geqq 0$ |  | $F_{1}(s)=\int_{0}^{T} f(t) e^{-s t} d t$ |

