

Name: KEY

Tables are attached

Given

$$e^{-|t|} \xleftrightarrow{\mathcal{F}} \frac{2}{\omega^2 + 1}$$

 Find the Fourier transform of the following $\frac{1}{2\pi(t^2+1)}$

By duality $F(t) = 2\pi f(-\omega)$

$$\Rightarrow \frac{2}{t^2 + 1} \xleftrightarrow{-1-\omega} 2\pi e^{-|t|}$$

By linearity $\frac{1}{2\pi(t^2+1)} \xleftrightarrow{\quad} \boxed{\frac{1}{2} e^{-|t|}}$

Consider a linear, time-invariant system with impulse response $h(t) = 0.5 \frac{\sin(2t)}{t}$

Find the system output if the input is $x(t) = \cos(t) + \sin(3t)$

$$= \frac{\sin(2t)}{2t}$$

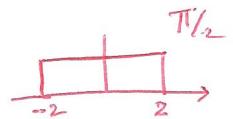
$$= \text{sinc}(2t)$$

Using Inverse Fourier Transform. $\frac{\beta}{\pi} \text{sinc}(\beta t) \leftrightarrow \text{rect}\left(\frac{\omega}{2\beta}\right)$

$$\text{sinc}(\beta t) \leftrightarrow \frac{\pi}{\beta} \text{rect}\left(\frac{\omega}{2\beta}\right)$$

only frequencies < 2 rad/sec will pass

$$\sin(2t) \leftrightarrow \frac{\pi}{2} \text{rect}\left(\frac{\omega}{4}\right)$$



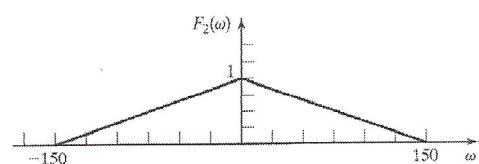
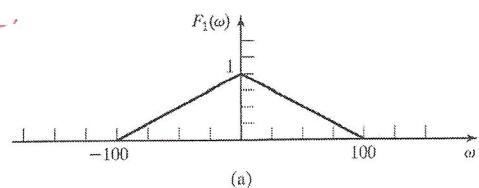
$$\boxed{y(t) = \frac{\pi}{2} \cos(t)}$$

The spectrum of two signals are shown in the Figure, write the relationship between the two signals in the time domain. i.e. write the relation between $f_1(t)$ and $f_2(t)$.

$$F_2(\omega) = F_1\left(\frac{\omega}{1.5}\right) \quad \text{By inspection.}$$

$$f_2(t) = \mathcal{F}^{-1} \left[\frac{1}{1.5} \frac{1}{|\omega|} F_1\left(\frac{\omega}{1.5}\right) \right]$$

$$\boxed{f_2(t) = 1.5 f_1(1.5t)}$$



Time Scaling. $f(a\omega) \leftrightarrow \frac{1}{|a|} F\left(\frac{\omega}{a}\right)$

The first two parts are HW problems
Good luck, Dr. Ali Muqaibel