Serial #

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- 1 points for not writing your serial number

King Fahd University of Petroleum & Minerals
Electrical Engineering Department
EE207: Signals and Systems (111)
Quiz 5: Laplace Transform

Name: Key Sec.

Find the Laplace transforms $F(s)$ of the following functions $f(t)$. Tables are attached.

$$f(t) = (1 - e^{-3t})u(t) = u(t) - e^{-3t}u(t)$$

$$F(s) = \frac{1}{s} - \frac{1}{s+3}$$

$$f(t) = (1 - 5e^{-2t})\delta(t) = \left(1 - 5e^{-2(0)}\right)\delta(t) = (1-5)\,\delta(t) = -4\delta(t)$$

$$F(s) = -4$$

$$f(t) = \sqrt{2}cos\left(100\pi t - \frac{\pi}{4}\right) = cos(100\pi t) + sin(100\pi t)$$

$$F(s) = \frac{s}{s^2 + (100\pi)^2} + \frac{100\pi}{s^2 + (100\pi)^2}$$

$$f(t) = \begin{cases} 0, & t < 2 \\ (t-2)^2, & t \geq 2 \end{cases}$$

$$f(t) = (t-2)^2 u(t-2)$$

Pair 3 with α=0

$$\frac{t^2 \exp(-\alpha t)\, u(t)}{n!} \leftrightarrow \frac{1}{(s+\alpha)^{n+1}}$$

$$\frac{t^2 u(t)}{2} \leftrightarrow \frac{1}{s^3}$$

$$t^2 u(t) \leftrightarrow \frac{2}{s^3}$$

$$F(s) = \frac{2e^{-2s}}{s^3}$$

Good Luck, **Dr. Ali Muqaibel**

TABLE 5-2
Laplace Transform Theorems

Name	Operation in Time Domain	Operation in Frequency Domain
1. Linearity	$a_1x_1(t) + a_2x_2(t)$	$a_1X_1(s) + a_2X_2(s)$
2. Differentiation	$\frac{d^nx(t)}{dt^n}$	$s^nX(s) - s^{n-1}x(0^-) - \cdots - x^{(n-1)}(0^-)$
3. Integration	$\int_{-\infty}^{t} x(\lambda)\, d\lambda$	$\frac{X(s)}{s} + \frac{x^{(-1)}(0^-)}{s}$
4. s-shift	$x(t)\exp(-\alpha t)$	$X(s+\alpha)$
5. Delay	$x(t-t_0)u(t-t_0)$	$X(s)\exp(-st_0)$
6. Convolution	$x_1(t) * x_2(t) = \int_0^{\infty} x_1(\lambda)x_2(t-\lambda)\, d\lambda$	$X_1(s)X_2(s)$
7. Product	$x_1(t)x_2(t)$	$\frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} X_1(s-\lambda)X_2(\lambda)\, d\lambda$
8. Initial value (provided limits exist)	$\lim_{t\to 0^+} x(t)$	$\lim_{s\to\infty} sX(s)$
9. Final value (provided limits exist)	$\lim_{t\to\infty} x(t)$	$\lim_{s\to 0} sX(s)$
10. Time scaling	$x(at), \quad a > 0$	$a^{-1}X\left(\frac{s}{a}\right)$

TABLE 5-3
Extended Table of Single-Sided Laplace Transforms

Signal	Laplace Transform	Comments on Derivation
1. $\delta^{(n)}(t)$	s^n	Direct evaluation with aid of (1-66)
2. 1 or $u(t)$	$\frac{1}{s}$	Direct evaluation
3. $\frac{t^n \exp(-\alpha t)u(t)}{n!}$	$\frac{1}{(s+\alpha)^{n+1}}$	Differentiation applied to pair 3, Table 5-1
4. $\cos \omega_0 t\, u(t)$	$\frac{s}{s^2+\omega_0^2}$	Example 5-1
5. $\sin \omega_0 t\, u(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	Example 5-1
6. $\exp(-\alpha t)\cos \omega_0 t\, u(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega_0^2}$	s-shift and pair 4
7. $\exp(-\alpha t)\sin \omega_0 t\, u(t)$	$\frac{\omega_0}{(s+\alpha)^2+\omega_0^2}$	s-shift and pair 5
8. Square wave: $u(t) - 2u\left(t - \frac{T_0}{2}\right) + 2u(t-T_0) - \cdots$	$\frac{1}{s}\frac{1-e^{-sT_0/2}}{1+e^{-sT_0/2}}$	Example 5-5
9. $(\sin \omega_0 t - \omega_0 t \cos \omega_0 t)u(t)$	$\frac{2\omega_0^3}{(s^2+\omega_0^2)^2}$	Example 5-12, pair 5, and convolution
10. $(\omega_0 t \sin \omega_0 t)u(t)$	$\frac{2\omega_0^2 s}{(s^2+\omega_0^2)^2}$	Pair 4 and convolution
11. $\omega_0 t \exp(-\alpha t)\sin \omega_0 t\, u(t)$	$\frac{2\omega_0^2(s+\alpha)}{[(s+\alpha)^2+\omega_0^2]^2}$	s-shift and pair 10
12. $\exp(-\alpha t)(\sin \omega_0 t - \omega_0 t \cos \omega_0 t)u(t)$	$\frac{2\omega_0^3}{[(s+\alpha)^2+\omega_0^2]^2}$	s-shift and pair 9

$\sin(x+y) = \sin x \cos y + \cos x \sin y,$
$\cos(x+y) = \cos x \cos y - \sin x \sin y,$

$\sin(x-y) = \sin x \cos y - \cos x \sin y,$
$\cos(x-y) = \cos x \cos y + \sin x \sin y.$

Good Luck, **Dr. Ali Muqaibel**