

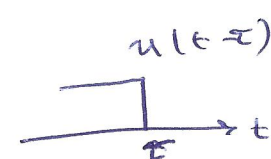
Name: KEY

Ver.2

Consider a system with impulse response $h(t) = u(t - 2)$

- a) Using the convolution integral, find the system response, $y(t)$, to the input,
 $x(t) = e^{-2t}u(t)$

$$\begin{aligned}
 y(t) &= \int_{-\infty}^{\infty} h(\tau) x(t-\tau) d\tau \\
 &= \int_{-\infty}^{\infty} u(\tau-2) e^{-2(t-\tau)} u(t-\tau) d\tau \\
 &= \int_2^{\infty} e^{-2(t-\tau)} u(t-\tau) d\tau
 \end{aligned}$$

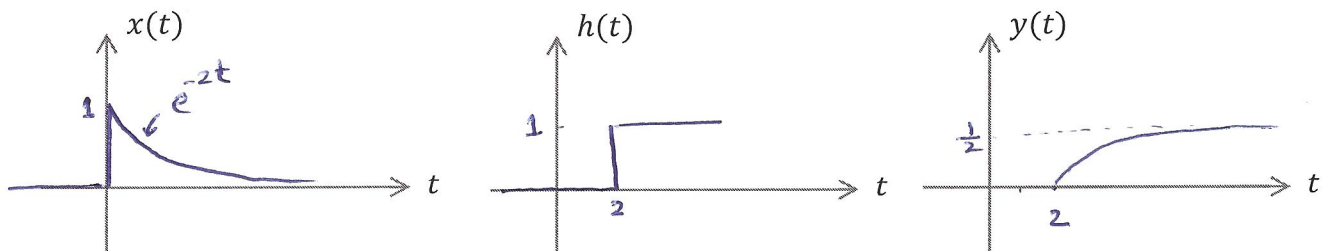


$$= \begin{cases} 0 & t < 2 \\ \int_2^t e^{-2(t-\tau)} d\tau & t > 2 \end{cases}$$

$$= \begin{cases} 0 & t < 2 \\ \frac{1}{2} e^{-2(t-\tau)} \Big|_2^t & t > 2 \end{cases}$$

$$= \begin{cases} 0 & t < 2 \\ \frac{1}{2} [1 - e^{-2(t-2)}] u(t-2) & t > 2 \end{cases}$$

- b) Sketch $x(t)$, $h(t)$ and $y(t)$. Discuss any indication from the sketch that $y(t)$ is correct!



start of $y(t)$ = start of $x(t)$ + start of $h(t)$ = $0 + 2 = 2$ ✓
 end + end = $\infty + \infty = \infty$ ✓

Area = $\infty \cdot \infty = \infty$
 Alternatively we can find the step response = $\int_{-\infty}^t h(\tau) d\tau$ & $s(t) = \frac{1}{2} [1 - e^{-2t}] u(t)$
 delay it by two units
 $s(t-2) = \frac{1}{2} [1 - e^{-2(t-2)}] u(t-2)$ ✓