King Fahd University of Petroleum & Minerals
Electrical Engineering Department
EE207: Signals and Systems (033)

Major Exam II

August 3, 2004
07:00 PM-08:30PM
Building 19-416

Serial #

Key
-2 points for not writing your serial #

Name:____________ **Key**
ID:________________________________

Sec. (1) 9:20-10:20 (2) 10:30-11:30

Question	Mark
1	/15
2	/15
3	/10
Total	/40

Instructions:
1. This is a closed-books/notes exam.
2. The duration of this exam is one and half hours.
3. Read the questions carefully. Plan which question to start with.
4. Write explicitly the formulas that you use in your solution (e.g. by KVL … by KCL). No credit will be given if you do not show your formulas.
5. Work in your own.
6. CLEARLY LABEL ALL SIGNIFICANT VALUES ON BOTH AXIES OF ANY SKETCH
7. Strictly no mobile phones are allowed.

Good luck

Dr. Ali Muqaibel

Problem 1: **(15 points)**

1. Find the Fourier Transform of the following signals (Tables are attached)
Indicate the property or pair number that you use **(6 points)**

a. $x_1(t) = \cos 10\pi t$

Using Table 4.2 pair #12

$\cos 2\pi f_0 t \longleftrightarrow \frac{1}{2}\delta(f-f_0) + \frac{1}{2}\delta(f+f_0)$

$\cos 2\pi(5)t \longleftrightarrow \frac{1}{2}\delta(f-5) + \frac{1}{2}\delta(f+5)$

b. $x_2(t) = \Lambda\left(\frac{t}{2}\right) + \exp(-3t)u(t)$

Using superposition Theorem & pairs 3 & 4

$\Lambda\left(\frac{t}{\tau}\right) \longleftrightarrow \tau\,\mathrm{sinc}^2\tau f$ $\quad \exp(-\alpha t)u(t), \alpha>0 \Leftrightarrow \frac{1}{\alpha+j2\pi f}$

$\Rightarrow X_2(f) = 2\,\mathrm{sinc}^2 2f + \frac{1}{3+j2\pi f}$

c. $x_3(t) = \delta(t)\cos(2\pi t)$

$\delta(t)\cos(2\pi t) = \delta(t)\cos(2\pi(0)) = \delta(t)$

property of $\delta(t)$, from pair #8 $\delta(t) \longleftrightarrow 1$

$X_3(f) = 1$

2. Given the following energy signal $x(t) = 2\exp(-2t)u(t)$**,**

a) What is the Fourier transform of this signal, $X(f)$? **(1 point)**

from the table similar to #15 $\quad X(f) = \frac{2}{2+j2\pi f} = \frac{1}{1+j\pi f}$

b) What is the energy spectral density, $G(f)$? **(1 point)**

$G(f) = |X(f)|^2 = \frac{1}{1+(\pi f)^2}$

c) What is the energy contained in the signal in the frequency range $-B<f<B$? **(3 points)**

replace variables $= \frac{2}{\pi}\int_0^{\pi B} \frac{1}{1+v^2}\,dv = \frac{2}{\pi}\left[\tan^{-1}\pi B - \tan^{-1} 0\right]$

because the function is even $\quad = \frac{2}{\pi}\tan^{-1}\pi B$

$$\boxed{\int \frac{1}{1+x^2}dx = \tan^{-1}x}$$

d) What is the total energy in the signal? **(2 points)**

Total Energy : $\lim_{B\to\infty} \frac{2}{\pi}\tan^{-1}\frac{B}{\pi} = \frac{2}{\pi}\tan^{-1}\infty = \frac{2}{\pi}\cdot\frac{\pi}{2} = 1$ Joule

e) Show that you can get the same total energy using the time domain definition of energy

$$E \,\square\, \lim_{T\to\infty}\int_{-T}^{T}|x(t)|^2\,dt = \int_{-\infty}^{\infty} G(f)\,df$$ **(2 points)**

$E = \lim_{T\to\infty}\int_{-T}^{+T}|x(t)|^2 dt = \int_{-\infty}^{\infty}(2\exp(-2t)u(t))^2 dt = 4\int_0^{\infty}\exp(-4t)\,dt = \frac{4}{-4}[0-1] = 1$ Joule

<u>Problem 2:</u> (15 points)

<u>Part I:</u>

a) Find the Laplace transform for the following signal by definition <u>(Laplace transform integral)</u>

(3 points)

$$x(f) = \int_0^\infty x(t)e^{-st}\,dt = 2\int_0^2 e^{-st}\,dt + \int_2^4 e^{-st}\,dt$$

$$= -\frac{2}{s}\left[e^{-2s} - 1\right] - \frac{1}{s}\left[e^{-4s} - e^{-2s}\right] = -\frac{2}{s}e^{-2s} + \frac{2}{s} - \frac{e^{-4s}}{s} + \frac{e^{-2s}}{s}$$

$$= \frac{1}{s}\left[2 - e^{-2s} - e^{-4s}\right]$$

b) Find the Laplace transform for the previous signal, by first representing it in terms of the step function *u*(*t*), and then using the provided tables for Laplace Transform **(3 points)**

$$x(t) = 2u(t) - u(t-2) - u(t-4)$$

$$X(f) = \frac{2}{s} - \frac{e^{-2s}}{s} - \frac{e^{-4s}}{s} = \frac{1}{s}\left[2 - e^{-2s} - e^{-4s}\right]$$

c) Show that the two answers in part **(a)** and **(b)** are equivalent (in case they have different forms, otherwise indicate that they are equivalent) **(1 point)**

They are already having the same form

<u>Part II :</u> Find the inverse Laplace transform for the following signals **(8 points)**

a) $Y_1(s) = \dfrac{16}{s^2 + 8s + 12}$

$$Y_1(s) = \frac{16}{s^2+8s+12} = \frac{16}{(s+6)(s+2)}$$

$$= \frac{A_1}{s+6} + \frac{A_2}{s+2}$$

$$A_1 = \frac{16}{s+2}\Big|_{s=-6} = \frac{16}{-4} = -4$$

$$A_2 = \frac{16}{s+6}\Big|_{s=-2} = \frac{16}{4} = 4$$

$$Y_1(s) = \frac{-4}{s+6} + \frac{4}{s+2}$$

$$y_1(t) = \mathcal{L}^{-1}[Y(s)] = (-4e^{-6t} + 4e^{-2t})u(t)$$

$$= 4(e^{-2t} - e^{-6t})u(t)$$

b) $Y_2(s) = \dfrac{10s}{(s+2)^2}$

$$Y_2(s) = \frac{10s}{(s+2)^2} = \frac{A_1}{s+2} + \frac{A_2}{(s+2)^2}$$

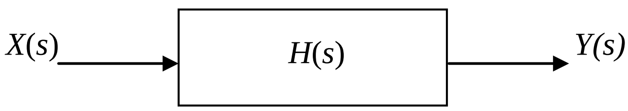
$$A_2 = Y_2(s)(s+2)^2\Big|_{s=-2} = 10(-2) = -20$$

$$A_1 = \frac{d}{ds}\left[(s+2)^2 Y(s)\right]\Big|_{s=-2} = \frac{d}{ds}(10s) = 10$$

$$\Rightarrow Y_2(s) = \frac{10}{s+2} - \frac{20}{(s+2)^2}$$

$$y_2(t) = \mathcal{L}^{-1}[Y_2(s)] = (10e^{-2t} - 20te^{-2t})u(t)$$

$$y_2(t) = 10e^{-2t}(1-2t)u(t)$$



<u>Problem 3:</u> (10 points)

1. Consider the following LTI system characterized by its transfer function $H(s)$:

<u>(1.5 point)</u> a) For a given input signal $X(s)$ and a transfer function $H(s)$, give the general expression of the output signal $y(t)$. (i.e How do we find the output $y(t)$?)

$$y(t) = \mathcal{L}^{-1}\left[X(s)H(s)\right]$$

<u>(1 point)</u> b) Write the general relation between the energy spectral density of the output signal $G_y(f)$, as function of the transfer function, $H(f)$, and the energy spectral density of the input signal, $G_x(f)$.

$$G_y(f) = |H(f)|^2 G_x(f)$$

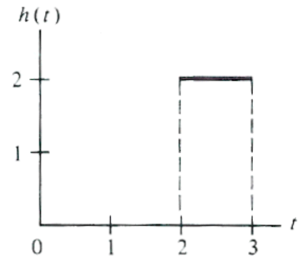
<u>(1.5 points)</u> c) Mention three advantages of using Laplace transform rather than Fourier transform?

1) More signals are transformable.

2) Can be used to find the transient response in addition to the steady state response.

3) Initial conditions are automatically included.

<u>(6 points)</u> d) <u>Using Laplace transformation (DO NOT perform graphical convolution),</u> Find out **and** <u>sketch</u>, y(t) which is given by the convolution of the shown input signal and impulse response, $y(t) = x(t) * h(t)$, show all details and important points on your sketch.



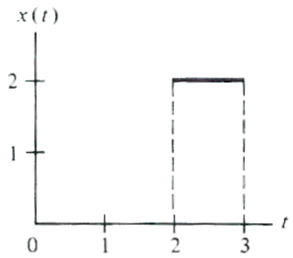
$x(t) = 2u(t-2) - 2u(t-3) = h(t)$

$X(s) = \frac{2}{s}\left[e^{-2s} - e^{-3s}\right] = H(s)$

$y(t) = x(t) \cdot y(t) \Rightarrow Y(s) = X(s)H(s)$

$Y(s) = \frac{2}{s}\left[e^{-2s} - e^{-3s}\right] \cdot \frac{2}{s}\left[e^{-2s} - e^{-3s}\right] = \frac{4}{s^2}\left[e^{-4s} - 2e^{-5s} + e^{-6s}\right]$

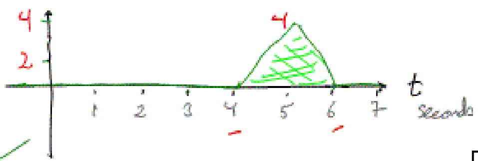
$y(t) = \mathcal{L}^{-1}[Y(s)] = 4r(t+4) - 8r(t-5) + 4r(t-6)$



Check start = 2 + 2 = 4 seconds ✓

end time = 3 + 3 = 6 seconds ✓

area = area$(h(t))$ · Area$(x(t))$ = 2.2 = 4 ✓



Laplace $(r(t)) = \frac{1}{s^2}$

Name:
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TABLE 4-1
Fourier Transform Theorems[a]

Name of Theorem		
1. Superposition (a_1 and a_2 arbitrary constants)	$a_1x_1(t) + a_2x_2(t)$	$a_1X_1(f) + a_2X_2(f)$
2. Time delay	$x(t - t_0)$	$X(f)e^{-j2\pi f t_0}$
3a. Scale change	$x(at)$	$\lvert a\rvert^{-1}X\left(\frac{f}{a}\right)$
b. Time reversal	$x(-t)$	$X(-f) = X*(f)$
4. Duality	$X(t)$	$x(-f)$
5a. Frequency translation	$x(t)e^{j\omega_0 t}$	$X(f - f_0)$
b. Modulation	$x(t) \cos \omega_0 t$	$\frac{1}{2}X(f - f_0) + \frac{1}{2}X(f + f_0)$
6. Differentiation	$\frac{d^n x(t)}{dt^n}$	$(j2\pi f)^n X(f)$
7. Integration	$\int_{-\infty}^{t} x(t')\, dt'$	$(j2\pi f)^{-1}X(f) + \frac{1}{2}X(0)\delta(f)$
8. Convolution	$\int_{-\infty}^{\infty} x_1(t - t')x_2(t')\, dt'$ $= \int_{-\infty}^{\infty} x_1(t')x_2(t - t')\, dt'$	$X_1(f)X_2(f)$
9. Multiplication	$x_1(t)x_2(t)$	$\int_{-\infty}^{\infty} X_1(f - f')X_2(f')\, df'$ $= \int_{-\infty}^{\infty} X_1(f')X_2(f - f')\, df'$

[a]$\omega_0 = 2\pi f_0$; $x(t)$ is assumed to be real in 3b.

TABLE 4-2
Fourier Transform Pairs

Pair Number	$x(t)$	$X(f)$	Comments on Derivation
1.	$\Pi\left(\frac{t}{\tau}\right)$	$\tau \operatorname{sinc} \tau f$	Direct evaluation
2.	$2W \operatorname{sinc} 2Wt$	$\Pi\left(\frac{f}{2W}\right)$	Duality with pair 1, Example 4-7
3.	$\Lambda\left(\frac{t}{\tau}\right)$	$\tau \operatorname{sinc}^2 \tau f$	Convolution using pair 1
4.	$\exp(-\alpha t)u(t),\ \alpha > 0$	$\frac{1}{\alpha + j2\pi f}$	Direct evaluation
5.	$t \exp(-\alpha t)u(t),\ \alpha > 0$	$\frac{1}{(\alpha + j2\pi f)^2}$	Differentiation of pair 4 with respect to α
6.	$\exp(-\alpha\lvert t\rvert),\ \alpha > 0$	$\frac{2\alpha}{\alpha^2 + (2\pi f)^2}$	Direct evaluation
7.	$e^{-\pi(t/\tau)^2}$	$\tau e^{-\pi(f\tau)^2}$	Direct evaluation
8.	$\delta(t)$	1	Example 4-9
9.	1	$\delta(f)$	Duality with pair 7
10.	$\delta(t - t_0)$	$\exp(-j2\pi f t_0)$	Shift and pair 7
11.	$\exp(j2\pi f_0 t)$	$\delta(f - f_0)$	Duality with pair 9
12.	$\cos 2\pi f_0 t$	$\frac{1}{2}\delta(f - f_0) + \frac{1}{2}\delta(f + f_0)$	Exponential representation of cos and sin and pair 10
13.	$\sin 2\pi f_0 t$	$\frac{1}{2j}\delta(f - f_0) - \frac{1}{2j}\delta(f + f_0)$	
14.	$u(t)$	$(j2\pi f)^{-1} + \frac{1}{2}\delta(f)$	Integration and pair 7
15.	$\operatorname{sgn} t$	$(j\pi f)^{-1}$	Pair 8 and pair 13 with superposition
16.	$\frac{1}{\pi t}$	$-j \operatorname{sgn}(f)$	Duality with pair 14
17.	$\hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\lambda)}{t - \lambda}\, d\lambda$	$-j \operatorname{sgn}(f)X(f)$	Convolution and pair 15
18.	$\sum_{m=-\infty}^{\infty} \delta(t - mT_s)$	$f_s \sum_{m=-\infty}^{\infty} \delta(f - mf_s),$ $f_s = T_s^{-1}$	Example 4-10

TABLE 5-2
Laplace Transform Theorems

Name	Operation in Time Domain	Operation in Frequency Domain
1. Linearity	$a_1x_1(t) + a_2x_2(t)$	$a_1X_1(s) + a_2X_2(s)$
2. Differentiation	$\dfrac{d^n x(t)}{dt^n}$	$s^nX(s) - s^{n-1}x(0^-) - \cdots - x^{(n-1)}(0^-)$
3. Integration	$\int_{-\infty}^{t} x(\lambda)\, d\lambda$	$\dfrac{X(s)}{s} + \dfrac{x^{(-1)}(0^-)}{s}$
4. s-shift	$x(t)\exp(-\alpha t)$	$X(s+\alpha)$
5. Delay	$x(t-t_0)u(t-t_0)$	$X(s)\exp(-st_0)$
6. Convolution	$x_1(t) * x_2(t) = \int_0^{\infty} x_1(\lambda)x_2(t-\lambda)\, d\lambda$	$X_1(s)X_2(s)$
7. Product	$x_1(t)x_2(t)$	$\dfrac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} X_1(s-\lambda)X_2(\lambda)\, d\lambda$
8. Initial value (provided limits exist)	$\lim_{t\to 0^+} x(t)$	$\lim_{s\to\infty} sX(s)$
9. Final value (provided limits exist)	$\lim_{t\to\infty} x(t)$	$\lim_{s\to 0} sX(s)$
10. Time scaling	$x(at), \quad a > 0$	$a^{-1}X\left(\dfrac{s}{a}\right)$

TABLE 5-3
Extended Table of Single-Sided Laplace Transforms

Signal	Laplace Transform	Comments on Derivation
1. $\delta^{(n)}(t)$	s^n	Direct evaluation with aid of (1-66)
2. 1 or $u(t)$	$\dfrac{1}{s}$	Direct evaluation
3. $\dfrac{t^n \exp(-\alpha t)u(t)}{n!}$	$\dfrac{1}{(s+\alpha)^{n+1}}$	Differentiation applied to pair 3, Table 5-1
4. $\cos\omega_0 t\, u(t)$	$\dfrac{s}{s^2+\omega_0^2}$	Example 5-1
5. $\sin\omega_0 t\, u(t)$	$\dfrac{\omega_0}{s^2+\omega_0^2}$	Example 5-1
6. $\exp(-\alpha t)\cos\omega_0 t\, u(t)$	$\dfrac{s+\alpha}{(s+\alpha)^2+\omega_0^2}$	s-shift and pair 4
7. $\exp(-\alpha t)\sin\omega_0 t\, u(t)$	$\dfrac{\omega_0}{(s+\alpha)^2+\omega_0^2}$	s-shift and pair 5
8. Square wave: $u(t) - 2u\left(t - \dfrac{T_0}{2}\right) + 2u(t-T_0) - \cdots$	$\dfrac{1}{s}\dfrac{1-e^{-sT_0/2}}{1+e^{-sT_0/2}}$	Example 5-5
9. $(\sin\omega_0 t - \omega_0 t\cos\omega_0 t)u(t)$	$\dfrac{2\omega_0^3}{(s^2+\omega_0^2)^2}$	Example 5-12, pair 5, and convolution
10. $(\omega_0 t\sin\omega_0 t)u(t)$	$\dfrac{2\omega_0^2 s}{(s^2+\omega_0^2)^2}$	Pair 4 and convolution
11. $\omega_0 t\exp(-\alpha t)\sin\omega_0 t\, u(t)$	$\dfrac{2\omega_0^2(s+\alpha)}{[(s+\alpha)^2+\omega_0^2]^2}$	s-shift and pair 10
12. $\exp(-\alpha t)(\sin\omega_0 t - \omega_0 t\cos\omega_0 t)u(t)$	$\dfrac{2\omega_0^3}{[(s+\alpha)^2+\omega_0^2]^2}$	s-shift and pair 9