King Fahd University of Petroleum & Minerals
Electrical Engineering Department
EE207: Signals and Systems (043)

Major Exam I

July 19, 2005
07:00 PM-08:30PM
Building 19-416

Serial #
0

-2 points for not writing your serial #

Name: KEY
ID:

Sec. (1) 9:20-10:20

Question	Mark
1	/18
2	/8
3	/14
Total	/40

Instructions:

1. This is a closed-books/notes exam.
2. The duration of this exam is one and half hours.
3. Read the questions carefully. Plan which question to start with.
4. Write explicitly the formulas that you use in your solution (e.g. by KVL … by KCL). No credit will be given if you do not show your formulas.
5. Work in your own.
6. CLEARLY LABEL ALL SIGNIFICANT VALUES ON BOTH AXIES OF ANY SKETCH
7. Strictly no mobile phones are allowed.

Good luck

Dr. Ali Muqaibel

Problem 1: (18 points)

1. State wither the following statements are True or False: (fill the table below with T or F)
+1 for any correct answer, and -0.5 for any wrong answer. Maximum=5, Minimum=0

- **a.** For a linear time invariant system; $a(t) = \int_{-\infty}^{\infty} h(t)dt$.
- **b.** Even functions are symmetric about the x-axis.
- **c.** The following system $y(t) = 5 + x(t) + x(t^2)$ is instantaneous.
- **d.** In the exponential Fourier representation, the complex coefficient X_n is real if the signal is half wave odd symmetric.
- **e.** $\delta(t)\cos(2\pi t) = \delta(t)$.

Q	a	b	c	d	e
T or F	F	F	F	F	T

2. Find the energy and the power of the following signal $x(t) = 2\exp(-2t)u(t)$. **(3 points)**

$$\text{Energy} = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} \left(2e^{-2t}u(t)\right)^2 dt = 4\int_0^{\infty} e^{-4t} dt = \frac{-4}{4}\left[e^{-4t}\right]_0^{\infty}$$

$$= -1\left[0 - 1\right] = 1$$ energy is finite $>0 \Rightarrow$ energy signal

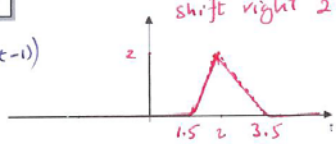
Power = zero $\boxed{E = 1}$ $\boxed{P = 0}$

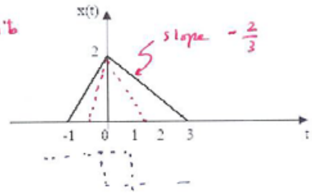
3. A continuous-time signal $x(t)$ is shown in the Figure. Sketch and label each of the following signals. **(6 points)**

i. $x(2t-4)$, $= x(2(t-2))$
ii. $x(t)u(1-t)$, $= x(t)u(-(t-1))$
iii. $x(t)\delta(t-2)$

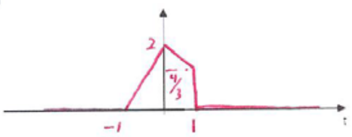
i

Compress factor of 2
shift right 2 unit



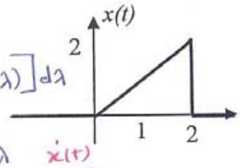


ii



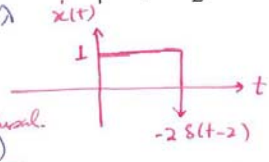
iii

4. A system has the following step response $a(t) = 4e^{-2t}u(t)$, what would be the output of the system if the input is $x(t)$ **(4 points)**
hint: you may use the convolutional integral in terms of step response



$$y(t) = \int_{-\infty}^{\infty} \dot{x}(\lambda)\, a(t-\lambda)\, d\lambda = \int_{-\infty}^{\infty} [u(\lambda) - u(\lambda-2) - 2\delta(\lambda-2)][4e^{-2(t-\lambda)}u(t-\lambda)]\, d\lambda$$

$$= \int_{-\infty}^{t} u(\lambda)\, 4e^{-2(t-\lambda)}\, d\lambda - \int_{-\infty}^{t} 4e^{-2(t-\lambda)} u(\lambda-2)\, d\lambda - 8\int_{-\infty}^{t} e^{-2(t-\lambda)}\delta(\lambda-2)\, d\lambda$$



$$= 4\int_0^t e^{-2(t-\lambda)} d\lambda - 4\int_2^t e^{-2(t-\lambda)} d\lambda - 8e^{-2(t-2)}$$

Causal.

$$= \frac{4}{+2}\left(1 - e^{-2t}\right)u(t) - \frac{4}{+2}\left[1 - e^{-2(t-2)}\right]u(t-2) - 8e^{-2(t-2)}u(t-2)$$

$\dot{x}(t) = u(t) - u(t-2) - 2\delta(t-2)$

$$= 2\left(1 - e^{-2t}\right)u(t) - 2\left[1 - e^{-2(t-2)}\right]u(t-2) - 8e^{-2(t-2)}u(t-2)$$

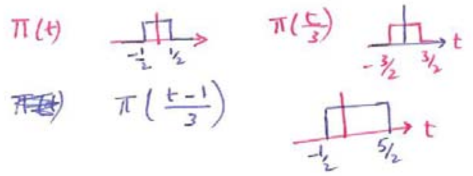
or by $x(t) = r(t) - r(t-2) - 2u(t-2)$ where $\gamma(t) = \int_{-\infty}^{t} a(\tau)\, d\tau$

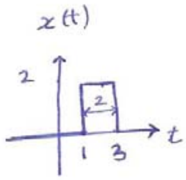
other solutions are possible with minor change in the results due to the discontinuity (def of u(t) @ t=0)

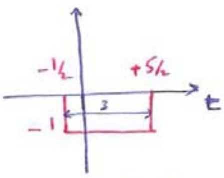
Problem 2: (8 points)

1. Perform convolution graphically to get the output. Show all steps and cases. Sketch the output signal $y(t) = x(t) * h(t)$

$x(t) = 2\prod\left(\frac{t-2}{2}\right)$, $h(t) = -\prod\left(\frac{t-1}{3}\right)$ →







The output will be limited to

$-\frac{1}{2}+1 = \frac{1}{2}$

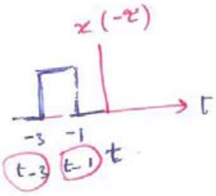
to

$3 + 5/2 = 11/2$

Area of output = $A_1 \cdot A_2$

$4 \cdot 3 = 12$

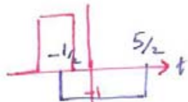
① No over lap



$-\infty < t < \frac{1}{2}$

② partial over lap.

$\frac{1}{2} \leq t < \frac{5}{2}$



$$\int_{-1/2}^{t-1} (2)(-1)\, dt = -2\left(t-1+\frac{1}{2}\right)$$

$= -2t+1$.

③ full over lap.

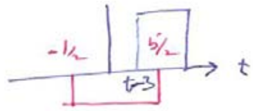
$\frac{5}{2} \leq t \leq \frac{7}{2}$

$$\int_{t-3}^{t-1} (-2)\, dt$$

$= -2(\cancel{t}-1-\cancel{t}+3) = -4$

④ partial over lap.

$\frac{7}{2} < t \leq \frac{11}{2}$



$$\int_{t-3}^{5/2} -2\, dt = -2\left[\frac{5}{2} - t + 3\right]$$

$= 2t - 11$

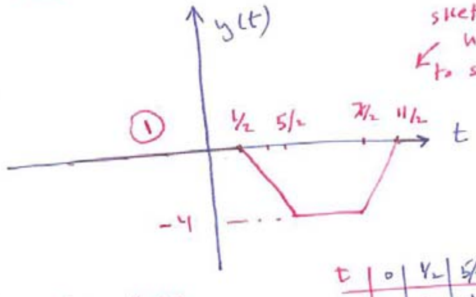
⑤ No over lap $\frac{11}{2} < t < \infty$

0

②

$$y(t) = \begin{cases} 0 & -\infty < t \leq \frac{1}{2} \\ -2t+1 & \frac{1}{2} \leq t \leq \frac{5}{2} \\ -4 & \frac{5}{2} \leq t \leq \frac{7}{2} \\ 2t-11 & \frac{7}{2} \leq t \leq \frac{11}{2} \\ 0 & \frac{11}{2} < t < \infty \end{cases}$$

0.5, 1, 1, 1, 0.5



sketch not to scale

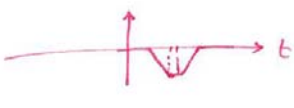
t	0	1/2	5/2	7/2	11/2
y(t)	0	0	-4	-4	0

Start = 1/2 (ok)

end = 11/2 (ok)

Area = $(\frac{7}{2}-\frac{5}{2})\cdot(4)$

$+\frac{1}{2}(\frac{5}{2}-\frac{1}{2})(4) = + 8 = 12$ (ok)



Problem 3: (15 points)

1. The complex Fourier series expansion of a periodic signal over an interval $0 \le t \le T$ is

$$x(t) = \sum_{n=-\infty}^{\infty} \frac{e^{2n}}{1 + j2\pi n} e^{j3.5\pi nt}$$

a) Determine numerical value of T. **(1 point)**

by comparing with $x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega_o t} \Rightarrow \omega_o = 3.5\pi \Rightarrow T = \frac{2\pi}{\omega_o} = \frac{2\pi}{3.5\pi}$

$T = 0.571$ s

b) What is the average value of $x(t)$ over one period. **(2 points)**

$= X_o = \frac{e^0}{1} = 1$

c) Determine the amplitude of the third harmonic component. **(1.5 points)**

$X_3 = \frac{e^6}{1 + j6\pi} = \quad |X_3| = 21.37 = |X_{-3}|$

d) Obtain the phase of the third harmonic component. **(1.5 points)**

$\angle X_3 = -\tan^{-1}\left(\frac{6\pi}{1}\right) = -86.96^\circ \quad \& \quad \angle X_{-3} = +86.96^\circ$

e) Determine the value of 3rd harmonic coefficients in the trigonometric Fourier series representation. a_3 and b_3 **(3 points)**

$X_n = \begin{cases} \frac{1}{2}(a_n - jb_n) & n>0 \\ \frac{1}{2}(a_{-n} + jb_{-n}) & n<0 \end{cases} \Rightarrow \quad X_3 + X_{-3} = a_3 = 0.0298$

$X_3 - X_{-3} = -jb_3 = 42.74$

f) Is $x(t)$ even ? Is it odd? *Justify your answer*. **(1 point)**

It is not odd & it is not even because X_n's are not pure real & are not pure imaginary.

2. Sketch the magnitude spectrum of the following signal. **(4 points)**

Hint: you might want to represent the signal in exponential form.

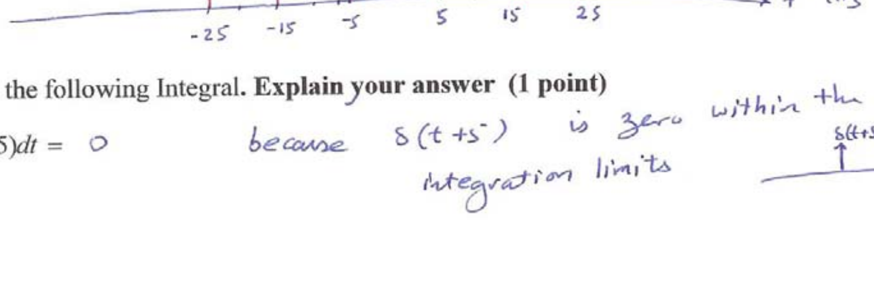
$$x(t) = \cos^2(20\pi t)\sin(10\pi t)$$

$x(t) = \left(\frac{e^{j20\pi t} - e^{-j20\pi t}}{2}\right)^2 \frac{e^{j10\pi t} - e^{-j10\pi t}}{2j}$

$= \frac{1}{j8}\left[e^{j40\pi t} - 2e^0 + e^{-j40\pi t}\right]\left[e^{j10\pi t} - e^{-j10\pi t}\right]$

$= \frac{-j}{8}\left[e^{j50\pi t} - 2e^{j10\pi t} + e^{-j30\pi t} - e^{+j30\pi t} + 2e^{-j10\pi t} - e^{-j50\pi t}\right]$

$=$



we can represent in terms of sin

3. Evaluate the following Integral. **Explain your answer (1 point)**

$\int_0^\infty e^{-2t}\delta(t+5)dt = 0$ because $\delta(t+5)$ is zero within the integration limits