King Fahd University of Petroleum & Minerals
Electrical Engineering Department
EE207: Signals and Systems (043)

Final Exam

Tuesday, August 23, 2005
12:30 PM-03:00PM
Building 14-108

Serial #
0

Name:
ID: KEY

Sec. (1) 9:20-10:20, Dr. Muqaibel (2) 10:30-11:30, Dr. Andalusi

Question	Mark
1	/10
2	/10
3	/10
4	/10
5	/10
Total	/50

Instructions:

1. This is a closed-books/notes exam.
2. Read the questions carefully. Plan which question to start with.
3. Write explicitly the formulas that you use in your solution (e.g. by KVL … by KCL). No credit will be given if you do not show your formulas.
4. Work in your own.
5. CLEARLY LABEL ALL SIGNIFICANT VALUES ON BOTH AXIES OF ANY SKETCH
6. Strictly no mobile phones are allowed.

Good luck

Dr. Ali Muqaibel & Dr. Adnan Andalusi

Problem 1: (10 points)

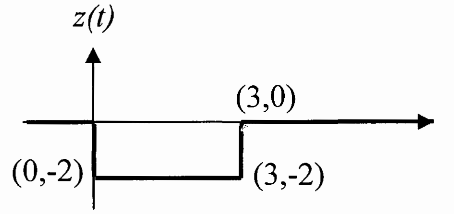
Choose (Circle) the correct answer (or) answers: **(5 points)**

a. If $y(t)$ is a real function of time then the magnitude of its Fourier transform is **(even, odd)** function of frequency.

b. The spectrum of a discrete time sampled signal is **(Periodic in frequency, non-periodic in frequency).**

c. The shown signal can be represented by

($-u(t)+u(t-3)$, $-2u(t)\times u(-t+3)$, $\Pi(\frac{t}{3}-0.5)$, $-2\,sinc\,(3t)$)



e. The signal $z(t)$, shown in the previous figure, is **(energy signal, power signal, neither energy nor power)**

d. A system which is represented by $y(t)=\cos(t)+x^2(t)$, where $x(t)$ is the input signal and $y(t)$ is the output signal is **(time varying, time invariant, linear, non-linear)**

(5 points)

2. State whether the following statements are True or False: (fill the table below with T or F) +1 for any correct answer, and -0.5 for any wrong answer. Maximum=5, Minimum=0

a. $\int_{-\infty}^{\infty} u(t)dt = r(t)$.

b. The ramp response of a causal system is zero for $t<0$.

c. The z-transform of the sequence $x[(n-1)T_s]$ is $z^{-1}X(z)$, where $X(z)$ is the z-transform of $x[n\,T_s]$.

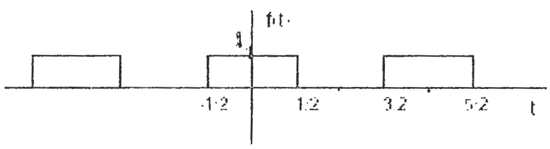
d. The convolution of two signals $x(t)$ and $y(t)$ in the time domain is equivalent to convolving $X(f)$ and $Y(f)$ in the frequency domain.

e. The bandwidth of a pulse is inversely proportional to its time duration.

Q	a	b	c	d	e
T or F	F	T	T	F	T

Problem 2: (10 points)

Consider the following periodic signal $f(t)$:



2 pts a) What is the period and fundamental frequency of this signal?

3 pts b) Show that the Fourier series of this signal is given by:

$f(t) = 1/2 + (2/\pi)*[\cos(\pi t) - 1/3 \cos(3\pi t) + 1/5 \cos(5\pi t) - 1/7 \cos(7\pi t) + \ldots]$

3 pts c) Give the exponential form of this Fourier series.

2 pts d) Give a simple plot of the double-sided line spectra of this signal (both magnitude and phase), and specify which one is even or odd.

a) $T_0 = 2\,s.$ (1), $f_0 = \frac{1}{2}\,Hz$ (1), $\omega_0 = \pi$ rad/sec

b) clearly $a_0 = \frac{1}{T_0}\int_{T_0} f(t)\,dt = 1/2$. (0.5)

$b_n = 0$ since $f(t)$ is even. (0.5)

$a_n = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} f(t)\cos n\omega_0 t\,dt$

(1) $= \frac{2}{2}\cdot 2\int_0^{1/2} 1\cdot\cos n\pi t\,dt = \frac{2}{n\pi}\sin n\pi t\Big|_0^{1/2} = \frac{2}{n\pi}\sin\left(\frac{n\pi}{2}\right)$

→ For n even, $a_n = 0$ ($\sin\frac{n\pi}{2} = 0$)

→ for n odd, $a_n = \pm\frac{2}{n\pi}$ ($\sin\frac{n\pi}{2} = \pm 1$)

(1) ↳ $a_n = \frac{2}{n\pi}$, $n = 1, 5, 9, 13, \ldots$

$= -\frac{2}{n\pi}$, $n = 3, 7, 11, 15, \ldots$

c) $f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}$

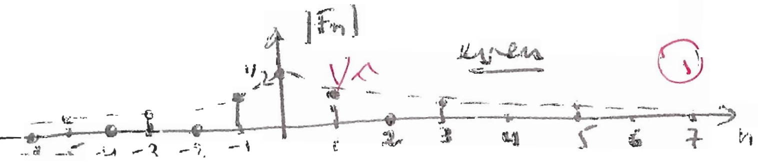
$F_0 = a_0$ (1), $F_n = \frac{1}{2}(a_n - jb_n)$ for $n \neq 0$. $F_0 = 1/2$

$F_{-n} = \frac{1}{2}(a_n + jb_n)$ (1)

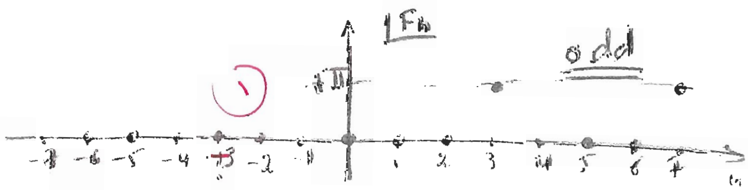
n odd → $F_n = \pm\frac{1}{n\pi}$ (1) $\left\{ +\frac{1}{n\pi}, n = 1, 5; \; -\frac{1}{n\pi}, n = 3, 7, \right.$

$F_{-n} = F_n$.

d) $|F_0| = \frac{1}{2}$, $|F_n| = \frac{1}{n\pi}$, n odd, $\angle F_n = \pm\pi$.



(1)



(1)

Problem 3: **(10 points)**

Consider the following signal which is sampled using ideal impulse train at a rate of 10 samples/second.

$$x(t) = 1 + 4\cos(4\pi t)$$

(1 point) a) Calculate the power of $x(t)$.

Power $= 1^2 + \frac{4^2}{2} = 1 + 8 = 9$

(1 point) b) For the first 4 samples fill in the following table:

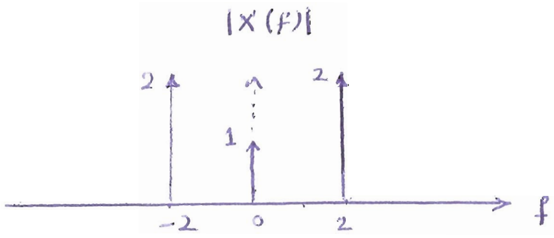
T_s $1/10$ seconds

	Sampling Time	*Sampled Value*
n	nT_s	$x(nT_s)$
0	0	5
1	0.1	2.236
2	0.2	-2.236
3	0.3	2.236

(3 points) c) Find and sketch the magnitude of $X(f)$, which represents the magnitude spectrum of $x(t)$.

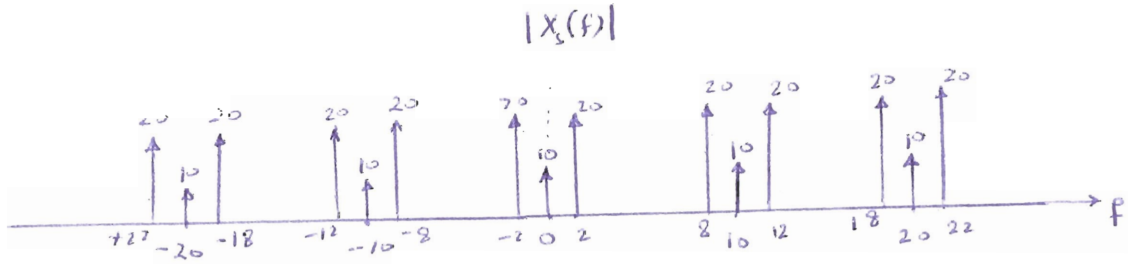
$X(f) = \delta(f) + 2\,\delta(f-2) + 2\,\delta(f+2)$

1



(3 points) d) Find and sketch $|X_s(f)|$, which represents the magnitude spectrum of the sampled signal $x_s(t)$, Your sketch should show the range of frequencies $-25<f<25$ Hz.

2 if no scale



(1 point) e) Suggest a method to reconstruct $x(t)$ from $x_s(t)$?

Use a low pass filter (ideal) with bandwidth (cut off) > 2 Hz & < 8 Hz

0.5 0.5

(1 point) f) What is the minimum required sampling frequency to avoid aliasing?

Sampling frequency should be $> 2f_M = 4$ Hz

Problem 4: (10 points)

Consider the following circuit:

2.5 1) Write the equivalent Laplace transform model for this circuit. You can assume that the initial conditions are all zero ($v_C(0)=0$ and $i_L(0)=0$).

2.5 2) Assume the output Y(s) is the voltage $V_R(s)$. Find the transfer function Y(s)/X(s).

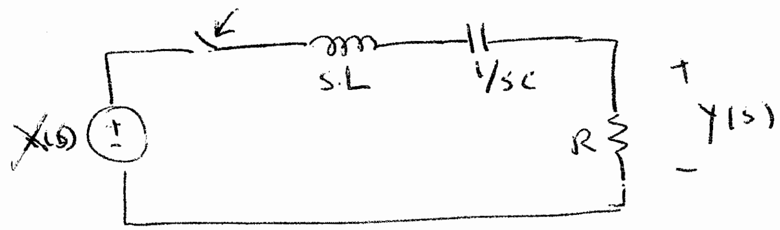
2.5 3) Assume that C=1F, L=1H, R=2Ω, and the input is a step: x(t) = u(t). Find the output function y(t).

2.5 4) Suppose now that C=0.125F, L=1H, R=6Ω, and the input is x(t)=exp(-t).u(t). Find the output function y(t). 1/8 F

Note: the following relationships are useful:

LT[exp(-αt).u(t)]= 1/(s+α) and LT[exp(-αt)sin(ω_ot).u(t)]=ω_o/[(s+α)^2 +ω_o^2]. ← remove!

1)



2)

$$Y(s) = R.I(s) = \frac{R\ X(s)}{sL + \frac{1}{sC} + R} = \frac{RC.s\ X(s)}{LC.s^2 + RC.s + 1}.$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{RC.s}{LC\,s^2 + RC.s + 1}$$

3)

$$x(t) = u(t) \longrightarrow X(s) = \frac{1}{s} \text{ and } Y(s) = \frac{2}{(s+1)^2}.$$

$$\longrightarrow y(t) = 2te^{-t}u(t).$$

4)

$$X(s) = \frac{1}{s+1} \text{ and } Y(s) = \frac{6s}{(s+1)(s^2+6s+8)} = \frac{6s}{(s+1)(s+2)(s+4)}$$

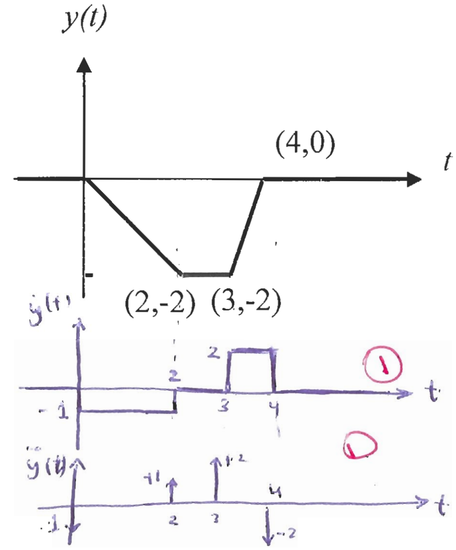
part. frac. exp. $\longrightarrow Y(s) = \frac{-2}{s+1} + \frac{6}{s+2} - \frac{4}{s+4}.$

and $y(t) = (-2e^{-t} + 6e^{-2t} - 4e^{-4t}).u(t)$

Problem 5: **(10 points)**

A. Find the Fourier transform of the following signals (Tables are attached, If needed)

Hint: you may use successive differentiation **(4 points)**



$\ddot{y}(t) = -\delta(t) + \delta(t-2) + 2\delta(t-3) - 2\delta(t-4)$

Using the tables

(1) $Y(f) = -1 + e^{-j4\pi f} + 2e^{-j6\pi f} - 2e^{-j8\pi f}$

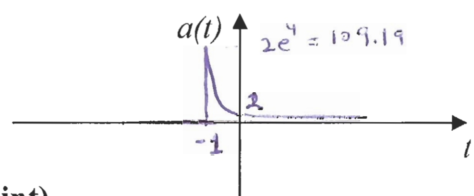
Using the differentiation property

$\frac{d^2}{dt^2} x(t) \Leftrightarrow (j2\pi f)^2 X(f)$

(1) $Y(f) = \frac{1}{(2\pi f)^2}\left[1 - e^{-j4\pi f} - 2e^{-j6\pi f} + 2e^{j8\pi f}\right]$

B. A linear fixed (time-invariant) system has the following **step** response **(6 points)**

$$a(t) = 2e^{-4t}u(t+1)$$



a. Sketch the step response as function of time. **(1 point)**

b. Is this system causal or non-causal? Justify your answer. **(1 point)**

No it is not causal because the output a(t) starts before the input u(t)

c. Find the impulse response, $h(t)$? **(1 point)** derivative of product

$h(t) = \frac{d}{dt} a(t) = \begin{cases} -8e^{-4t} & t > -1 \\ 0 & t < -1 \end{cases}$

or $-8e^{-4t}u(t+1) + 2e^{-4t}\delta(t+1)$

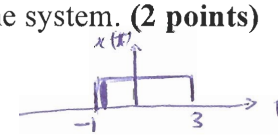
d. What is the output if the input is $x(t) = \delta(t-2) + 3u(t+4)$ **(1 point)**

Because the system is linear super position applies

$y(t) = h(t-2) + 3a(t+4) = -8e^{-4(t-2)}u(t-1) + 2e^{-4(t-2)}\delta(t-1)$

$+ 6e^{-4(t+4)}u(t+5)$

e. If the input is now given by $x(t) = \Pi\left(\frac{t-1}{4}\right)$, what would be the output of the system. **(2 points)**



$x(t) = u(t+1) - u(t-3)$

$\Rightarrow y(t) = a(t+1) - a(t-3)$

$2e^{-4(t+1)}u(t+2) - 2e^{-4(t-3)}u(t-2)$