# King Fahd University of Petroleum \& Minerals Electrical Engineering Department EE 207 - Signals and Systems 

## Final Examination

June 8, 2005

## Time Allowed $21 / 2$ Hours

Student Name (CAPITAL) : $\qquad$
Student ID Number : $\qquad$
Student Serial Number : $\qquad$

1. NO CALCULATORS ARE ALLOWED
2. SHOW ALL WORK
3. Answer 5 Problems from the 6 problems only

| Problem | Max Score | Score |
| :---: | :---: | :---: |
| Problem 1 | 20 |  |
| Problem 2 | 20 |  |
| Problem 3 | 20 |  |
| Problem 4 | 20 |  |
| Problem 5 | 20 |  |
| Problem 6 | 20 |  |
| Total | 100 |  |

## Problem 1 (20pts):

Figure 1 shows a non-linear system that consists of a squarer, and a linear timeinvariant system whose transfer function $H(f)$ is shown in fig. 2. If the input signal $x(t)=1+2 \sin (30 \pi t)$ is applied to the system, find

1. the complex Fourier coefficients of $x(t)$.
2. the complex Fourier coefficient of $v(t)\left(v(t)=x^{2}(t)\right)$
3. the output $y(t)$.
4. The average powers in
a. $x(t)$
b. $v(t)$
c. $y(t)$


Figure 1: Non-linear system


Figure 2: Amplitude \& phase spectrum of the linear system.

## Problem 2 (20pts):

A certain system has its impulse response $h(t)=\frac{1}{2 W} \operatorname{sinc}\left(2 \mathrm{~W}\left(\mathrm{t}-\mathrm{t}_{0}\right) \cos \left(2 \pi f_{0} t\right)\right.$.
Use Fourier transform properties to :

1. Find the transfer function $H(f)$ of the system.
2. Draw the amplitude spectrum $|H(f)|$
3. Draw the phase spectrum $\angle H(f)$

## Problem 3 ( 20 pts )

The signal
$x(t)=3+4 \cos (10 \pi t)+5 \cos (14 \pi t)+2 \cos (20 \pi t)$
is sampled at a rate of 30 samples per second. Sketch the spectrum of the sampled signal showing all components for $|f|<80$. Fully explain how $\mathrm{x}(\mathrm{t})$ can be reconstructed from the samples.

## Problem 4 (20 pts)

For the circuit below, the switch is in open position before $t=0$, then closed instantaneously at $\mathrm{t}=0$.

The initial capacitor voltage is 50 V , and the initial inductor current is 2 A .
Determine the current across the capacitor, $i(t)$, using s-domain analysis.


## Problem 5 (20 pts)

1. Determine the inverse Laplace transform of each of the following functions:

$$
X(s)=\frac{16 s+43}{(s-2)(s+3)^{2}}
$$

2. Determine the Laplace transform of the following function (hint: use the Laplace Transform properties)


## Problem 6 ( 20 pts)

A linear fixed (time-invariant) system has the following step response

$$
a(t)=e^{-2 t} u(t+1)
$$

a. Sketch the step response as function of time.
b. Is this system causal or non-causal? Justify your answer.
c. Find the impulse response, $h(t)$ ?
d. What is the output if the input is $x(t)=\delta(t-2)+5 u(t+5)$
e. If the input is now given by $x(t)=\Pi\left(\frac{t-1}{4}\right)$, what would be the output of the system.

Hint: recall that $\Pi(t)$ is the unit pulse function.

