King Fahd University of Petroleum and Minerals

Department of Electrical Engineering
EE 205 Circuit II,

Major Exam II
Saturday, 23 May 2009
6:45 - 8:45 PM

Name: KEY

ID: 000

Serial #: __________
Section: __________

Instructor: ______________________________

Problem	Score	Out of
1		14
2		10
3		10
4		14
Total		48

Good luck,

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Grading Guideline
→ 1 point for the relation
→ 1 point for the final answer

Problem 1: [14 points]

A. Two parallel RLC Circuits have the <u>same resonant frequency</u> **(2 points)**

Circuit 1: has quality factor, $Q_1 = 30$, $BW_1 = 10\ kHz$

Circuit 2: has quality factor, $Q_2 = 20$, $BW_2 = ?$ *Find the unknown bandwidth*

$$Q = \frac{\omega_r}{BW} \Rightarrow \omega_r = Q \cdot BW$$

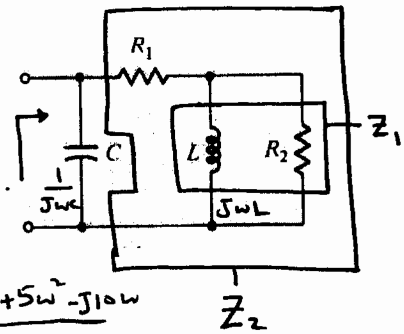
Since the two circuit have the same resonance frequency $\omega_{r_1} = \omega_{r_2} \Rightarrow$

$$Q_1 \cdot BW_1 = Q_2 \cdot BW_2 \Rightarrow BW_2 = \frac{Q_1 \cdot BW_1}{Q_2} = \frac{30}{20}\ 10\,K\,Hz = \boxed{15\ KHz}$$

B. Find the resonance frequency for the given circuit <u>in Hz,</u>

Let L=1 H, C=10.13 pF, R_1=3 Ohms, R_2=2 Ohms. **(6 points)**

Low or No credit if you do not explain your steps



Z_1 is made of $R_2 \parallel L$ $\boxed{1\ \text{point}}$

$$Z_1 = \frac{J\omega L R_2}{J\omega L + R_2} = \frac{J\,2\omega}{J\omega + 2}$$

substitute R_2 & L

Z_2 is made up of $Z_1 + R_1$ $\boxed{1\ \text{point}}$

$$Z_2 = R_1 + \frac{J\,2\omega}{J\omega + 2}$$

$$= 3 + \frac{J2\omega}{J\omega + 2} = \frac{3J\omega + 6 + J2\omega}{J\omega + 2}$$

$$= \frac{6 + J5\omega}{2 + J\omega}$$

$$Y_2 = \frac{1}{Z_2} = \frac{2 + J\omega}{6 + J5\omega}$$ $\boxed{1\ \text{point}}$

Y_{total} is made up of $Y_2 \parallel C$

$$Y_{total} = J\omega c + \frac{2 + J\omega}{6 + J5\omega} \cdot \frac{6 - J5\omega}{6 - J5\omega}$$

multiply by the conjugate

$\boxed{1\ \text{point for the final Expression}}$ Z total.

$$Y_{total} = J\omega c + \frac{J6\omega + 12 + 5\omega^2 - J10\omega}{25\omega^2 + 36}$$

to find the resonance frequency $\boxed{0.5p}$

Imaginary $(Y_{total}) = 0$ $\omega \leftarrow \omega_r$

$$\cancel{\omega_r} C + \frac{6\,\cancel{\omega_r} - 10\,\cancel{\omega_r}}{25\omega_r^2 + 36} = 0$$

divid ω_r $(\omega_r \neq 0)$

$$C - \frac{4}{25\omega_r^2 + 36} = 0$$ $\boxed{1\ \text{point}}$

$$\Rightarrow 25\omega_r^2 + 36 = \frac{4}{C}$$

$$\Rightarrow \omega_r^2 = \frac{1}{25}\left(\frac{4}{C} - 36\right)$$

$$\omega_r = \frac{1}{5}\sqrt{\frac{4}{C} - 36} = 125.62K \text{ rad/sec.}$$

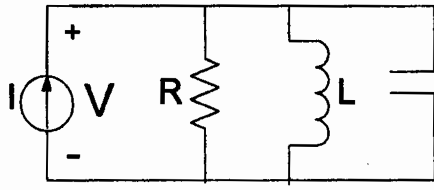
$$\boxed{f_r = \frac{\omega_r}{2\pi} = 20\ KHz = 20\,000\ Hz}$$

$\boxed{0.5\ \text{point}}$

1.C

Show that the Quality factor at resonance of the parallel RLC circuit is equal to $Q = \omega_r RC$,

hint $\left(Q = 2\pi \frac{[w_C(t) + w_L(t)]_{max}}{P_R T} \right)$ **(6 points)**



No credit if you do not explain your steps

— Let $i(t) = I\cos w_r t$

then the Parallel Voltage is

1 — $v(t) = R\,i(t) = RI\cos w_r t$

Thus, the energy stored in the capacitor is

$w_C(t) = \frac{1}{2} C v^2(t)$ — 0.25

0.5 —— $= \frac{1}{2} C R^2 I^2 \cos^2 w_r t$ — 0.25

— The inductor current is

$I_L = \frac{RI\angle 0^\circ}{w_r L \angle 90^\circ} = \frac{RI}{w_r L}\angle -90^\circ$

$\Rightarrow i_L(t) = \frac{RI}{w_r L}\cos(w_r t - 90^\circ)$

1 —— $= \frac{RI}{w_r L}\sin w_r t$

$\Rightarrow$ Energy stored in the inductor is $\frac{1}{2} L i^2$ 0.25

0.5 — $W_L(t) = \frac{1}{2} C R^2 I^2 \sin^2 w_r t$ 0.25

— $w_C(t) + W_L(t) = \frac{1}{2} C R^2 I^2 \cos^2 w_r t + \frac{1}{2} C R^2 I^2 \sin^2 w_r t$

$= \frac{1}{2} C R^2 I^2$

1 — $[w_C(t) + w_L(t)]_{max} = \frac{1}{2} C R^2 I^2$

— The power absorbed by the resistor is

$P_R = \frac{1}{2} R I^2$ — 1

$\Rightarrow P_R T = \frac{1}{2} I^2 R T$ 0.5

$= \frac{1}{2} I^2 R \left(\frac{2\pi}{w_r}\right)$

$= \frac{\pi I^2 R}{w_r}$

— $\therefore$ $Q = \frac{2\pi\left(\frac{1}{2} C R^2 I^2\right)}{\pi I^2 R / w_r}$

$\boxed{Q = w_r R C}$ — 1

Grading Guidelines.

1 p: Voltage on the capacitor.

0.5 p: Energy on " "

1 p: Current on the coil.

0.5: Energy stored in the coil.

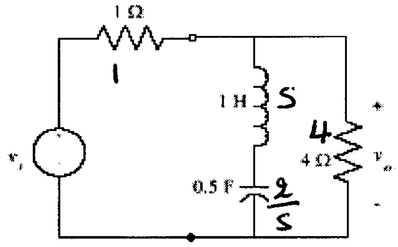
1 p: max Energy stored.

1 p: Power dissipated in a period in R.

1 p: Simplifying Q to the required Expression.

Problem 2: [10 points]

Consider the following circuit:



a) Find the expression of the transfer function $V_o(s)/V_s(s)$ **[4 points]**

$$\frac{V_o}{V_s} = \frac{4 \,//\, (s + \frac{2}{s})}{1 + \{4 \,//\, (s+\frac{2}{s})\}} \quad 1pt$$

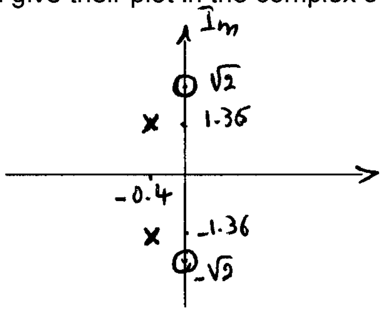
$$= \frac{\dfrac{4 \times (s + 2/s)}{4 + s + 2/s}}{1 + \dfrac{4 \times (s + 2/s)}{4 + s + 2/s}}$$

$$\frac{V_o}{V_s} = \frac{4s^2 + 8}{5s^2 + 4s + 10} \quad 3pt$$

b) Find the poles and zeros of this transfer function, and give their plot in the complex s-plane (use appropriate notation) **[2 points]**

Zeros : $4s^2 + 8 = 0 \rightarrow s = \pm j\sqrt{2}$ 0.5pt

poles : $5s^2 + 4s + 10 = 0 \rightarrow s = -0.4 \pm j\,1.356$ 0.5pt



1pt

c) What is the type of the natural response in the time domain? **[1 points]**

& decaying (damped) sin. of the form $A_1 e^{(-0.4+j1.36)t} + A_2 e^{(-0.4+j1.36)t}$ or $B_1 e^{-0.4t}\cos 1.36t + B_2 e^{-0.4t}\sin 1.36t$

↳ stable!

d) Suppose the input voltage is $v_s(t) = 10e^{-t}\cos(2t + 30^0)V$. Find the step forced steady state output response $v_o(t)$. **[3 points]**

$v_s(t) = 10e^{-t}\cos(2t + 30°) \rightarrow V_s = 10\angle 30$ 0.25pt

$s = -1 + 2j$ 0.25pt

$$V_o = \frac{4(-1+2j)^2 + 8}{5(-1+2j)^2 + 4(-1+2j) + 10} \times 10\angle 30°$$

$$= \frac{-4 - 16j}{-9 - 12j} \times 10\angle 30°$$

$$\cong 1.1\angle 22.8° \times 10\angle 30°$$

$\Rightarrow v_o(t) = 11 \cos(2t + 52.8°)$

0.5pt

Solution

Problem 3: [10 points]

For the transformer in the figure,

L_1= 25 mH and L_2= 100 mH.

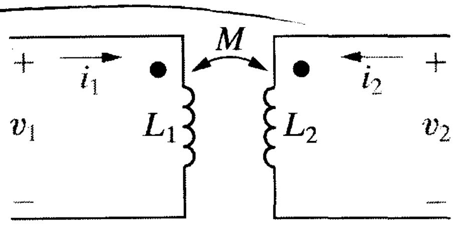


Figure 06-30

3.1 If the coefficient of coupling (*k*) is equal to 0.8,

a) Find the mutual inductance M **[1 point]**

$M = k\sqrt{L_1 L_2}$ = 0.5

$M = 0.8\sqrt{(0.025)(0.1)} = 40\,\text{mH}$ — 0.5

b) Find the turn ratio N_2/N_1, assuming that the two coils have the same permeance. **[1 point]**

0.5 — $L_1 = P_1 N_1^2$ and $L_2 = P_2 N_2^2$ | Since $P_1 = P_2$ $\Rightarrow \frac{L_2}{L_1} = \frac{N_2^2}{N_1^2} \Rightarrow \frac{N_2}{N_1} = \sqrt{\frac{L_2}{L_1}}$ (0.5) $= \sqrt{\frac{100}{25}} = 2$

c) Find the energy stored in the system when i_1 = -10 A, i_2 = 15 A **[3 point]**

1 — $W = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2$

$= \frac{1}{2}(0.025)(-10)^2 + \frac{1}{2}(0.1)(15)^2 + \underbrace{(0.04)(-10)(15)}_{1} = \underbrace{6.5\ \text{J}}_{1}$

3.2 Find the value of *k* that makes the mutual inductance M = 112 mH (justify your answer) **[2 point]**

$M = k\sqrt{L_1 L_2} = k(50)\,\text{mH} \Rightarrow$ k should be 2.24

However since $\underbrace{0 \leq k \leq 1}_{1} \Rightarrow$ we $\underbrace{\text{can't}}_{1}$ have a mutual inductance = 112 mH

3.3 If the coefficient of coupling (*k*) is equal to 1 and If i_1 = 10 A, find the value of i_2 that makes the energy stored in the systems equals zero. **[3 point]**

when $k = 1 \Rightarrow M = 1\sqrt{(0.025)(0.1)} = \underbrace{50\,\text{mH}}_{1}$, $i_1 = 10\,\text{A}$

1 — $\Rightarrow 50 i_2^2 + 500 i_2 + 1250 = 0$

$\Rightarrow i_2^2 + 10 i_2 + 25 = 0$

$\Rightarrow i_2 = \frac{-10 \pm \sqrt{10^2 - 4(25)}}{2}$

1 — $\boxed{i_2 = -5\ \text{A}}$

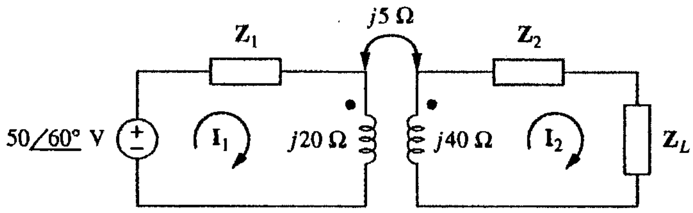
$J\omega M \Rightarrow \omega M = 5$

Problem 4: [14 Points]

For the circuit shown in the Figure,

Let **Z_1 = 60-*j*100 Ω, Z_2= 30+*j*40 Ω,**

and **Z_L= 80+*j*60 Ω,**



A. Calculate the impedance seen by the ideal source $\left(Z_{int} = \frac{V_s}{I_1}\right)$ **[3 points]**

We can do (B) first

$$Z_{int} = Z_1 + j20 + Z_r$$
$$= 60 - j100 + j20 + Z_r$$
$$= 60 - j80 + 0.0868 - j0.114$$
$$= 60.0868 - j\,80.114$$

B. Find the reflected Impedance (Z_r) as seen from the primary side. **[3 points]**

$$Z_r = \frac{\omega^2 M^2}{|Z_{22}|^2} Z_{22}^* \quad , \omega M = 5$$

$$Z_r = \frac{(5)^2}{31700}(110 - j140)$$

$$= 0.0868 - j0.114 \quad \Omega$$

$$Z_{22} = 30 + j40 + 80 + j60 + j40$$
$$= 110 + j140$$
$$= 178.05 \angle 51.843^\circ$$

Note: Z_r little impedance is reflected. This due to the relatively low mutual Inductance $\underline{\underline{j5}}$.

C. Find the mesh currents *I_1* and *I_2*. **[4 points]**

$$I_1 = \frac{V_s}{Z_{int}} = \frac{50 \angle 60^\circ}{Z_{in}}$$

$$= 0.499 \angle 113.13^\circ \text{ A}$$

By KVL at loop "secondary.

$$I_2 Z_{22} - J5\, I_1 = 0$$

$$\Rightarrow I_1 = \frac{I_2 Z_{22}}{5J}$$

$$I_2 = \frac{J5\, I_1}{Z_{22}} = \frac{5 \angle 9^\circ\, I_1}{178.05 \angle 51.843^\circ}$$

$$= 0.014 \angle 151.287^\circ \text{ A}$$

C. Calculate the power consumed by the load $\mathbf{Z_L}$. **[2 points]**

Assuming the given voltage is RMS.

$$I^2 R = (0.014)^2 (80)$$
$$= 15.68 * 10^{-3} \text{ Watts}$$

if the given voltage is amplitude

$$P = \frac{1}{2} I^2 R$$
$$= \frac{1}{2} I^2 R = 7.84 * 10^{-3} \text{ Watts}$$

D. If the given circuit is shown for a radian frequency of **5** rad/sec, find the mutual inductance (*M*) and the coupling coefficient (*k*). **[2 points]**

$$j\omega M = j5$$

$$\Rightarrow \omega M = 5 \quad \Rightarrow \quad M = \frac{5}{5} = 1 \text{ H}$$

Similarly

$$j\omega L_1 = j20 \quad \Rightarrow \quad L_1 = 4 \text{ H}$$
$$j\omega L_2 = j40 \quad \Rightarrow \quad L_2 = 8 \text{ H}$$

$$M = K\sqrt{L_1 L_2} \quad \Rightarrow \quad K = \frac{1}{\sqrt{4 \cdot 8}} = 0.1768$$