

King Fahd University of Petroleum & Minerals  
Electrical Engineering Department  
EE205: Electric Circuits II (031)

## Quiz 5

Name: KEY II assume no formulas

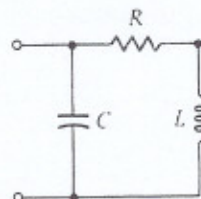
ID#

Sec 01

For the practical tank circuit shown in the figure

$$R=0.002 \Omega, L=2 \text{ H, and } C=\frac{1}{50} \text{ F}$$

- Find the resonance frequency of the circuit
- Find the quality factor of the circuit



$$\begin{aligned} a) \quad Y_{in} &= j\omega C + \frac{1}{R + j\omega L} = j\omega C + \frac{R - j\omega L}{R^2 + \omega^2 L^2} \\ &= \frac{R}{R^2 + \omega^2 L^2} + j\left(\omega C - \frac{\omega L}{R^2 + \omega^2 L^2}\right) \end{aligned}$$

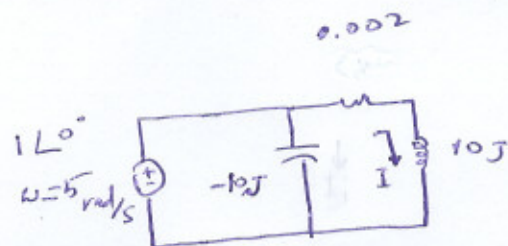
To find  $\omega_r$  we let the imaginary part = 0.

$$\omega C - \frac{\omega L}{R^2 + \omega^2 L^2} = 0 \Rightarrow R^2 + \omega_r^2 L^2 = \frac{L}{C} \Rightarrow \omega_r^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\omega_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = \sqrt{\frac{1}{2\left(\frac{1}{50}\right)} - \frac{4 \times 10^{-6}}{4}} = \sqrt{25 - 10^{-6}} \approx 5 \text{ rad/s}$$

b) Quality factor

$$I = \frac{1 \angle 0^\circ}{0.002 + j\omega L} = 0.1 \angle -90^\circ = -0.1j$$



$$W_C(t) = \frac{1}{2} C v^2(t) = \frac{1}{2} \left(\frac{1}{50}\right) \cos^2 5t = \frac{1}{100} \cos^2 5t$$

$$W_L(t) = \frac{1}{2} L i^2(t) = \frac{1}{2} (2) \left(\frac{1}{10}\right)^2 \sin^2 5t = \frac{1}{100} \sin^2 5t$$

$$W_C(t) + W_L(t) = \frac{1}{100} (\cos^2 5t + \sin^2 5t) = \frac{1}{100} \quad W_{max} = \frac{1}{100}$$

$$P_R = \frac{1}{2} |I|^2 R = \frac{1}{2} (0.1)^2 (0.002) = 10^{-5} \quad T = \frac{2\pi}{\omega} = \frac{2\pi}{5} \text{ s}$$

$$Q = 2\pi \left( \frac{W_{max}}{P_R T} \right) = 2\pi \left( \frac{\frac{1}{100}}{10^{-5} \frac{2\pi}{5}} \right) = 10^{-2} * 10^5 * 5 = 5000$$