Serial #

King Fahd University of Petroleum & Minerals
Electrical Engineering Department
EE205: Electric Circuits II (031)
Quiz 3

Name: KEY ID# Sec. 02

The initial value of the voltage v in the circuit Fig. 8.1 is zero, and the initial value of the capacitor current, $i_c(0^+)$ is 10mA. The expression for the capacitor current is known to be

$$i_c(t) = A_1 e^{-80t} + A_2 e^{-20t} \qquad , t \geq 0^+,$$

when R is 100Ω. Find the numerical value of $\alpha, \omega_0, L, C, A_1, A_2$

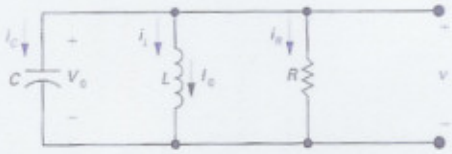

Figure 8.1 A circuit used to illustrate the natural response of a parallel *RLC* circuit.

The response is overdamped

$i_c(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$

$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$

$s_1 + s_2 = -2\alpha \Rightarrow -80 - 20 = -2\alpha \Rightarrow \boxed{\alpha = 50 \text{ nepers}}$

$s_1 - s_2 = 2\sqrt{\alpha^2 - \omega_0^2} \Rightarrow -\frac{60}{2} = \sqrt{2500 - \omega_0^2} \Rightarrow 900 = 2500 - \omega_0^2$

$\Rightarrow \omega_0^2 = 1600 \Rightarrow \boxed{\omega_0 = 40 \text{ rad/s}}$

$\alpha = \frac{1}{2RC} \Rightarrow C = \frac{1}{2\alpha R} = \frac{1}{2(50)100} = \boxed{100\,\mu F = C}$

$\omega_0^2 = \frac{1}{LC} \Rightarrow L = \frac{1}{\omega_0^2 C} = \boxed{6.25\ H = L}$

$i_c(0^+) = \boxed{A_1 + A_2 = 10\text{ mA}}$ ①

for the inductor $v_L = L\frac{di}{dt} \Rightarrow \frac{di(0^+)}{dt} = \frac{v(0^+)}{L} = 0$

$\frac{di_R(0)}{dt} = \frac{1}{R}\frac{dv(0)}{dt} = \frac{1}{R}\frac{i_c(0)}{C} = \frac{10 \times 10^{-3}}{(100)(100\,\mu F)} = 1\ A/s$

$i_c = -i_L - i_R \Rightarrow \frac{di_c(0^+)}{dt} = -\frac{di_L(0^+)}{dt} - \frac{di_R(0^+)}{dt} = -1$

$\therefore +80A_1 + 20A_2 = +1 \Rightarrow \boxed{4A_1 + A_2 = 50\text{ mA}}$ ②

Solving ① & ②

$\Rightarrow \boxed{A_1 = 13.33\text{ mA}}$

$\boxed{A_2 = -3.33\text{ mA}}$

Good Luck,
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