

King Fahd University of Petroleum & Minerals
 Electrical Engineering Department
 EE205: Electric Circuits II (031)

Quiz 3

Name: **KEY**

ID#

Sec. 01

The initial value of the voltage v in the circuit Fig. 8.1 is zero, and the initial value of the capacitor current, $i_c(0^+)$ is 10mA. The expression for the capacitor current is known to be

$$i_c(t) = A_1 e^{-80t} + A_2 e^{-20t}, \quad t \geq 0^+,$$

when R is 100Ω . Find the numerical value of $\alpha, \omega_0, L, C, A_1, A_2$

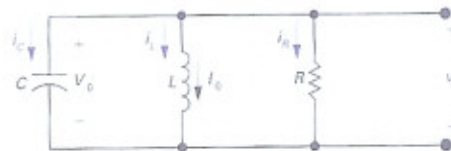


Figure 8.1 A circuit used to illustrate the natural response of a parallel RLC circuit.

The response is overdamped

$$i_c(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

$$s_1 + s_2 = -2\alpha \Rightarrow -80 - 20 = -2\alpha \Rightarrow \boxed{\alpha = 50 \text{ nepers}}$$

$$s_1 - s_2 = 2\sqrt{\alpha^2 - \omega_0^2} \Rightarrow -\frac{60}{2} = \sqrt{2500 - \omega_0^2} \Rightarrow 900 = 2500 - \omega_0^2$$

$$\Rightarrow \omega_0^2 = 1600 \Rightarrow \boxed{\omega_0 = 40 \text{ rad/s}}$$

$$\alpha = \frac{1}{2RC} \Rightarrow C = \frac{1}{2\alpha R} = \frac{1}{2(50)100} = \boxed{100 \mu\text{F} = C}$$

$$\omega_0^2 = \frac{1}{LC} \Rightarrow L = \frac{1}{\omega_0^2 C} = \boxed{6.25 \text{ H} = L}$$

$$i_c(0^+) = \boxed{A_1 + A_2 = 10 \text{ mA}} \quad \text{①, for the inductor } v_L = L \frac{di}{dt} \Rightarrow \frac{di(t)}{dt} = \frac{v(t)}{L} = 0$$

$$\frac{di_c(0^+)}{dt} = \frac{1}{R} \frac{dv(0^+)}{dt} = \frac{1}{R} \frac{i_c(0^+)}{C} = \frac{10 \times 10^{-3}}{(100)(100 \mu\text{F})} = 1 \text{ A/s} \quad \text{Solving ① \& ②}$$

$$i_c = -i_L - i_R \Rightarrow \frac{di_c(0^+)}{dt} = -\frac{di_L(0^+)}{dt} - \frac{di_R(0^+)}{dt} = -1$$

$$\therefore +80A_1 + 20A_2 = +1 \Rightarrow \boxed{4A_1 + A_2 = 50 \text{ mA}} \quad \text{②}$$

$$\boxed{\begin{matrix} A_1 = 13.33 \text{ mA} \\ A_2 = -3.33 \text{ mA} \end{matrix}}$$