# Computer Aided Circuit Analysis and State Variable Analysis

**Ver. 2.1 Prepared by <u>Dr. Ali Hussein Muqaibel</u>**

- One way of solving complicated circuits containing inductors, capacitors is through computer aided circuit analysis.
- The order of the circuit $n$ is equivalent to the number of non-trivial inductors and capacitors.
- The variable of interest (voltage or current) can be represented by single $n^{th}$ order differential equation or $n$ first order equations.
- Computer aided circuit analysis involves two Major steps:
  - Convert the circuit to matrix state equation. (**a.** systematic, **b.** non-systematic)
  - Numerically solve the matrix state equation.

### 1.a Convert the circuit to matrix state equation (non-systematic).

- The conversion can be done in a non-systematic way to come up with $n$ first order equations.
  - Usually to find $\frac{dv_C}{dt}$, we apply KCL at a relevant node.
  - Usually to find $\frac{di_L}{dt}$, we apply KVL in the relevant loop.
- For a circuit of the 2 order the matrix state equation may have the following form. If no source exists the second term in the right-hand is zero

$$\frac{\mathrm{d}}{\mathrm{dt}}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} w(t)$$

It's better you check the equations after writing them by multiplication property of the matrices. A common mistake is to miss-place the elements of the matrix

### 1.b Convert the circuit to matrix state equation (systematic).

- A systematic procedure is needed to be programmed on a computer.
- You need to know some Graph Theory concepts (Graph, node, edges, connected graph, spanning tree, branch, non-branch, loop, fundamental loop, cut set, fundamental cut set).

**<u>Important terms in GRAPH Theory</u>**

***GRAPH*** **:** is a collection of junction points called ***NODES***, and line segments connecting the nodes called ***BRANCHES***

***TREE*** **:** a connected sub-graph, connecting all nodes of the graph but containing no loops. Branches of a tree are called **TREE-Branches**. Other Branches are called **NON-TREE-BRANCHES**.

***Spanning Tree*** **:** a tree chosen to write circuit equations.

***Cut set*** **:** a minimum set of branches that, when cut, will divide a graph into two separate parts.

***Fundamental cut set*** **:** a cut set containing only a single tree branch.

***Fundamental loop*** **:** a loop that results when a link is put into the tree.

## Formal Procedure for Obtaining State Equations

**STEP1:** Pick a spanning tree such that voltage sources and capacitors correspond to branches, whereas current sources and inductors correspond to non-branch edges. Furthermore, if possible, an element whose voltage controls a dependent source should correspond to a branch, and one whose current controls a dependent source should correspond to a non-branch edge. More than one such tree may exist, or none at all.

**STEP2:** Arbitrarily assign a voltage to each branch capacitor and a current to each non-branch inductor, these are the state variables. If possible, express the voltage across each element corresponding to a branch and the current through each element corresponding to a non-branch edge in terms of voltage sources, current sources, and state variables. If it is not possible, assign a new voltage variable to a resistor corresponding to a branch and new current variable to a resistor corresponding to a non-branch edge.

**STEP3:** Apply *KVL* to the fundamental loop determined by each non-branch inductor.

**STEP4:** Apply *KCL* to the node or super-node corresponding to the fundamental cut-set determined by each branch capacitor.

**STEP5:** Apply *KVL* to the fundamental loop determined by each resistor with a new current variable assigned in STEP2.

**STEP6:** Apply *KCL* to the node or super-node corresponding to the fundamental cut-set determined by each resistor with a new voltage variable assigned in STEP2.

**STEP7:** Solve the simultaneous equations obtained from STEP5 and STEP6 for the new variables in terms of the voltage sources, current sources, and state variables.

**STEP8:** Substitute the expressions obtained in STEP7 into the equations determined in STEP3 and STEP4.

**2. Numerically solve the matrix state equation.**

- **Utilizing Euler's Method:** $x([k+1]\Delta t) = x(k\Delta t)[1 + A\ \Delta t]$
- The matrix state equation can be solved numerically:
- For a two circuit of the second order and a single source, the solution is given by

$$x_1([k+1]\Delta t) = x_1(k\Delta t) + [\, a_{11}x_1(k\Delta t) + a_{12}x_2(k\Delta t) + b_1 w(k\Delta t)]\,\Delta t\,.$$

$$x_2([k+1]\Delta t) = x_2(k\Delta t) + [\, a_{21}x_1(k\Delta t) + a_{22}x_2(k\Delta t) + b_2 w(k\Delta t)]\,\Delta t\,.$$

- where : $(a_{11}, a_{12}, a_{21}, a_{22})$ are the component of the matrix.
- $k = 0, 1, 2, 3, \ldots\ldots$ , $b1$ and $b_2$ are component of B matrix (given source $w(t)$)
- The initial conditions *for* $x_1$ and ,$x_2$ should be given /can be calculated.
- The previous equation can be easily extended to higher order circuits and more sources.