Resonance

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- Resonance happens when the imaginary part of the *impedance* or *admittance* seen by the source is equal to zero. The frequency at which resonance occur is denoted, ω_r .
- Not all circuits have real resonance frequency.
- Quality factor is measured at resonance and defined to be

$$Q = 2\pi \frac{[\text{maximum energy stored}]}{[\text{total energy lost in a period}]} = 2\pi \frac{[W_L(t)+W_C(t)]_{\max}}{P_R T}$$

T is the period and is given by $T = \frac{1}{f} = \frac{2\pi}{\omega_r}$.

- The cutoff frequency is the frequency at which the output (voltage /current) its maximum divided by $\sqrt{2}$. Remember that the maximum occurs at resonant frequency, ω_r . Usually there are two cutoff frequencies, ω_1 and ω_2 .
- The bandwidth, BW= $\omega_2 - \omega_1$.
- Selectively is defined to be the ration of resonance frequency to BW, $selectivity = \frac{\omega_r}{BW}$

	Series RLC	Parallel RLC
Q	$\frac{\omega_r L}{R}$	$\omega_r RC$
at ω_r *magnitude of*	impedance is minimum	admittance is minimum
ω_r	$1/\sqrt{LC}$	
ω_1 and ω_2	$\omega_1 = -\frac{\omega_r}{2Q} + \omega_r \sqrt{\left(\frac{1}{2Q}\right)^2 + 1}$, $\omega_2 = \frac{\omega_r}{2Q} + \omega_r \sqrt{\left(\frac{1}{2Q}\right)^2 + 1}$	
BW	$\frac{\omega_r}{Q}$	

- For series and parallel RLC circuits the natural frequency, $\omega_n = \frac{1}{\sqrt{LC}}$, equals to the resonant frequency. This is generally not true for other circuits.
- Quality Factor of Series and Parallel Reactances

$$Q = \frac{|X_s|}{R_s} \quad , \quad Q = \frac{R_p}{|X_p|}$$

For a practical tank circuit, we prove that Q for the inductor branch is high, then we can convert the circuit into equivalent parallel RLC circuit with new $R = L/R_s C$. Equations for parallel RLC circuits can be used to find the quality factor or resonance frequency (caution use new *R*)

- Magnitude/Impedance Scaling and Frequency Scaling

Original Circuit	Magnitude/Impedance Scaling	Frequency Scaling
R	$R' = k_m R$	$R'' = R$
C	$C' = C/k_m$	$C'' = C/k_f$
L	$L' = k_m L$	$L'' = L/k_f$