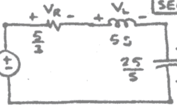
SECTION 10.4

10.40



$V_1 = 20\angle 0°$

$s = -6 + j3$

(a) $$V_R = \frac{5/3}{5/3 + 5s + 25/s} V_1$$

$$V_R = \frac{5s V_1}{5s + 15s^2 + 75}$$

$$V_R = \frac{5(-6+j3)(20)}{5(-6+j3) + 15(-6+j3)^2 + 75} = \frac{-600 + j300}{-30 + j15 + 15(36 - j36 - 9) + 75}$$

$$= \frac{-300(2-j)}{-30 + j15 + 405 - j540 + 75} = \frac{-300(2-j)}{450 - j525} = \frac{-20(2-j)}{30 - j35} = \frac{-20(2-j)}{5(6-j7)}$$

$$= \frac{(4\angle 180°)(\sqrt{5}\angle -26.565°)}{\sqrt{85}\angle -49.4°} = 0.97\angle -157.17°$$

$\therefore v_R(t) = 0.97e^{-6t}\cos(3t - 157.17°)$ V

(b) $$V_L = \frac{5s V_1}{\frac{5}{3} + 5s + \frac{25}{s}} = \frac{15s^2 V_1}{5s + 15s^2 + 75}$$

$$= \frac{15(-6+j3)^2(20)}{5(-6+j3) + 15(-6+j3)^2 + 75} = \frac{300(36 - j36 - 9)}{450 - j525} = \frac{300(27 - j36)}{75(6-j7)} = \frac{36(3-j4)}{(6-j7)}$$

$$= \frac{36(5\angle -53.13°)}{(\sqrt{85}\angle -49.4°)} = 19.52\angle -3.73°$$

$\therefore v_L(t) = 19.52e^{-6t}\cos(3t - 3.73°)$ V

(c) $$V_c = \frac{\frac{25}{s} V_1}{\frac{5}{3} + 5s + \frac{25}{s}} = \frac{75 V_1}{5s + 15s^2 + 75} = \frac{75(20)}{75(6-j7)} = \frac{20}{\sqrt{85}\angle -49.4°}$$

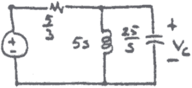
$V_c = 2.17\angle 49.4°$

$\therefore v_c(t) = 2.17e^{-6t}\cos(3t + 49.4°)$ V

10.42

$V_1 = 20\angle 0^\circ$

$s = -6 + j3$



By KCL,

$$\frac{V_c - V_1}{5/3} + \frac{V_c}{5s} + \frac{V_c}{25/s} = 0$$

$$\frac{3V_c - 3V_1}{5} + \frac{V_c}{5s} + \frac{sV_c}{25} = 0$$

$$15sV_c - 15sV_1 + 5V_c + s^2 V_c = 0$$

$$(s^2 + 15s + 5)V_c = 15sV_1$$

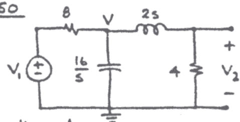
$$V_c = \frac{15sV_1}{s^2 + 15s + 5}$$

$$V_c = \frac{15(-6+j3)(20)}{(-6+j3)^2 + 15(-6+j3) + 5} = \frac{-900(2-j)}{36 - j36 - 9 - 90 + j45 + 5} = \frac{-900(2-j)}{-58 + j9}$$

$$= \frac{-900(2-j)}{-1(58 - j9)} = \frac{900(\sqrt{5}\angle -26.565^\circ)}{\sqrt{3445}\angle -8.82^\circ} = 34.29\angle -17.745^\circ$$

$$\therefore v_c(t) = 34.29e^{-6t}\cos(3t - 17.745^\circ)\ \text{V}$$

10.50



By KCL at node V,

$$\frac{V-V_1}{8} + \frac{V}{16/s} + \frac{V-V_2}{2s} = 0$$

$$2sV - 2sV_1 + s^2V + 8V - 8V_2 = 0$$

$$(s^2+2s+8)V - 8V_2 = 2sV_1$$

By voltage division,

$$V_2 = \frac{4}{2s+4}V \Rightarrow V = \frac{2s+4}{4}V_2 \Rightarrow (s^2+2s+8)\left(\frac{2s+4}{4}V_2\right) - 8V_2 = 2sV_1$$

$$(s^2+2s+8)(s+2)V_2 - 16V_2 = 4sV_1$$

$$(s^3+2s^2+8s+2s^2+4s+16-16)V_2 = 4sV_1$$

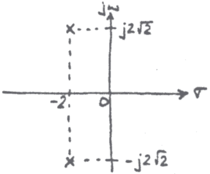
$$(s^3+4s^2+12s)V_2 = 4sV_1$$

$$(s^2+4s+12)V_2 = 4V_1$$

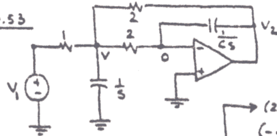
$$\therefore H(s) = \frac{V_2}{V_1} = \frac{4}{s^2+4s+12}$$

$$s = \frac{-4 \pm \sqrt{16-48}}{2} = -2 \pm \sqrt{\frac{-32}{4}} = -2 \pm \sqrt{-8} = -2 \pm j2\sqrt{2}$$

$$H(s) = \frac{4}{(s+2+j2\sqrt{2})(s+2-j2\sqrt{2})}$$



10.53



By KCL at inverting input,

$$\frac{V_2}{1/Cs} + \frac{V}{2} = 0$$

$$-2CsV_2 = V$$

$$(2s+4)(-2CsV_2) - V_2 = 2V_1$$

$$(-4Cs^2 - 8Cs - 1)V_2 = 2V_1$$

$$\therefore H(s) = \frac{V_2}{V_1} = \frac{-2}{4Cs^2 + 8Cs + 1}$$

By KCL at node V,

$$\frac{V - V_1}{1} + \frac{V}{1/s} + \frac{V}{2} + \frac{V - V_2}{2} = 0$$

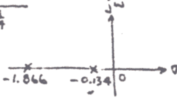
$$2V - 2V_1 + 2sV + V + V - V_2 = 0$$

$$(2s+4)V - V_2 = 2V_1$$

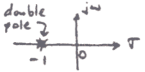
(a) $C = 1F \Rightarrow H(s) = \dfrac{-2}{4s^2 + 8s + 1} = \dfrac{-1/2}{s^2 + 2s + \frac{1}{4}}$

$$s = \frac{-2 \pm \sqrt{4-1}}{2} = -1 \pm \frac{\sqrt{3}}{2} = -1 \pm 0.866$$

$$H(s) = \frac{-1/2}{(s + 0.134)(s + 1.866)}$$



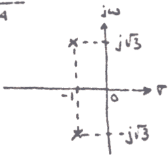
(b) $C = \frac{1}{4}F \Rightarrow H(s) = \dfrac{-2}{s^2 + 2s + 1} = \dfrac{-2}{(s+1)^2}$



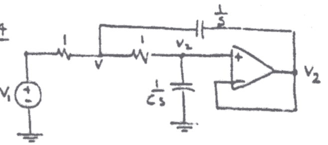
(c) $C = \frac{1}{16}F \Rightarrow H(s) = \dfrac{-2}{\frac{1}{4}s^2 + \frac{1}{2}s + 1} = \dfrac{-8}{s^2 + 2s + 4}$

$$s = \frac{-2 \pm \sqrt{4 - 16}}{2} = -1 \pm j\sqrt{3}$$

$$H(s) = \frac{-8}{(s + 1 + j\sqrt{3})(s + 1 - j\sqrt{3})}$$



10.54



At node V,

$$\frac{V-V_1}{1}+\frac{V-V_2}{1}+\frac{V-V_2}{1/s}=0$$

$$V-V_1+V-V_2+sV-sV_2=0$$

$$(s+2)V-V_1-(s+1)V_2=0 \Rightarrow (s+2)(Cs+1)V_2-(s+1)V_2=V_1$$

By voltage division,

$$V_2=\frac{\frac{1}{Cs}}{\frac{1}{Cs}+1}V=\frac{1}{1+Cs}V$$

$$V=(1+Cs)V_2$$

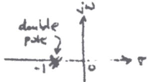
$$(Cs^2+2Cs+s+2-s-1)V_2=V_1$$

$$\therefore H(s)=\frac{V_2}{V_1}=\frac{1}{Cs^2+2Cs+1}=\frac{1/C}{s^2+2s+1/C}$$

(a) $C=\frac{1}{2}F \Rightarrow \frac{V_2}{V_1}=\frac{2}{s^2+2s+2}$

$$\frac{-2\pm\sqrt{-4}}{2}=-1\pm j \qquad =\frac{2}{(s+1-j)(s+1+j)}$$

(b) $C=1F \Rightarrow \frac{V_2}{V_1}=\frac{1}{s^2+2s+1}=\frac{1}{(s+1)^2}$



(c) $C=2F \Rightarrow \frac{V_2}{V_1}=\frac{\frac{1}{2}}{s^2+2s+\frac{1}{2}}$

$$\frac{-2\pm\sqrt{2}}{2}=-1\pm\frac{1}{\sqrt{2}}$$

$$-1\pm 0.707 \rightarrow -0.293, -1.707$$