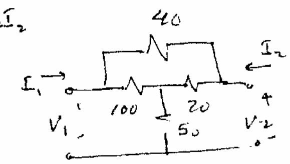
18-4

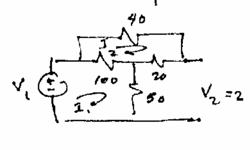
$V_1 = b_{11} V_2 - b_{12} I_2$

$I_1 = b_{21} V_2 - b_{22} I_2$



$b_{11} = \frac{V_1}{V_2} \Big|_{I_2=0} \longrightarrow$ port ② open circuit

So



$V_1 = 150 I_1 - 100 I_2$ --①

$0 = 160 I_2 - 100 I_1$ --②

$\downarrow$

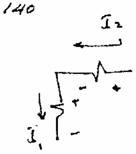
$\therefore I_1 = 1.6 I_2$

Subst. in ①:

$V_1 = (150 \times 1.6 - 100) I_2 = 140 I_2$

$\therefore I_2 = \frac{V_1}{140}$ $\qquad I_1 = \frac{1.6}{140} V_1$

$\therefore V_2 = 20 I_2 + 50 I_1$



$V_2 = \left(\frac{20}{140} + \frac{50 \times 1.6}{140}\right) V_1$

$V_2 = \frac{100}{140} V_1$ $\qquad \therefore \frac{V_1}{V_2}\Big|_{I_2=0} = 1.4$.

$\therefore b_{11} = 1.4$.

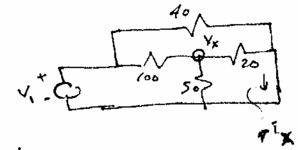
Also $\quad b_{21} = \frac{I_{in}}{V_2}\Big|_{I_2=0}$ $\qquad I_{in}$ here $= I_1$

$\therefore b_{21} = \frac{1.6}{140} \frac{V_1}{V_2} = \frac{1.6}{140} \times 1.4 = 1.6 \times 10^{-2}$

$= 0.016$

Next,

$b_{12} = -\frac{V_1}{I_2}\Big|_{V_2=0} \rightarrow$ port 2 short circuit



KCL at V_x:

$\frac{V_x - V_1}{100} + \frac{V_x}{50} + \frac{V_x}{20} = 0$

$V_x\left(\frac{1}{100} + \frac{1}{50} + \frac{1}{20}\right) = \frac{V_1}{100}$

$V_x \frac{1+2+5}{100} = \frac{V_1}{100} \rightarrow 8V_x = V_1$

$\therefore V_x = (V_1/8)$.

$\therefore I_x = \frac{V_x}{20} + \frac{V_1}{40} = V_1\left(\frac{1}{160} + \frac{1}{40}\right)$

$I_x = V_1 \frac{5}{160}$ $\qquad$ but $I_x = -I_2$

$\therefore b_{12} = +\frac{160}{5} = +32$

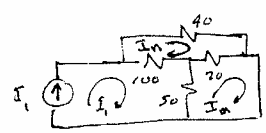
Finally, since the circuit is reciprocal

$\therefore b_{11} b_{22} - b_{12} b_{21} = 1$

$\therefore 1.4 b_{22} - 16 \times 10^{-3} \times 32 = 1$

$1.4 b_{22} = 1.512$ $\qquad \therefore b_{22} = 1.08$

Also, b_{22} may be obtained from the relation: $b_{22} = -\frac{I_1}{I_2}\Big|_{V_2=0}$ — Port 2 short circ



$I_m = -I_2$

KVL(I_m): $70 I_m - 20 I_n = 50 I_1$ ---①

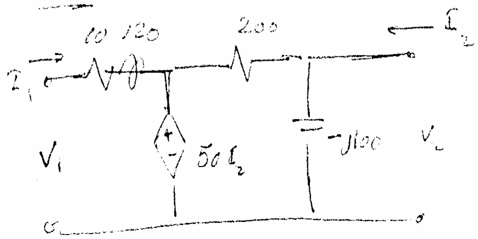
$160 I_n - 20 I_m = 100 I_1$ ---②

Solving ① & ②:

$540 I_m = 500 I_1$

$\therefore \frac{I_1}{I_m} = -\frac{I_1}{I_2} = b_{22} = \frac{540}{500} = 1.08$

18-10

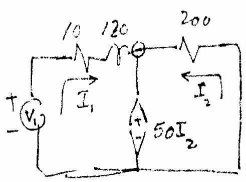


h-parameters:

$V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

$h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0} \rightarrow$ port 2 is short circuit



KVL (~~left~~ Right h. loop)

$\therefore 50 I_2 = -200 I_2$

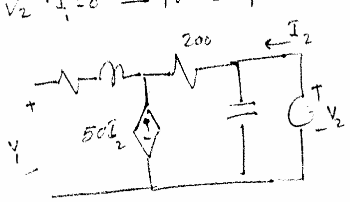
$\therefore I_2 = 0.$

$\therefore I_1 = \frac{V_1}{10+j20}$ $\quad\therefore \frac{V_1}{I_1}\Big|_{V_2=0} = (10+j20)$

Next, $h_{21} = \frac{I_2}{I_1}\Big|_{V_2=0}$ but $I_2 = 0$ when $V_2 = 0$ as shown earlier

$\therefore h_{21} = 0$

$h_{12} = \frac{V_1}{V_2}\Big|_{I_1=0} \rightarrow$ Port 1 open circuit



obviously $V_1 = 50 I_2$ --- ①

Using KCL:

$$\frac{V_2}{-j100} + \frac{V_2 - 50 I_2}{200} = I_2$$

$$\therefore V_2 \left(\frac{1}{200} - \frac{1}{j100}\right) = (1.25) I_2$$

$$\therefore I_2 = \frac{V_2}{1.25}\left(\frac{1}{200} - \frac{1}{j100}\right) \text{ --- ②} \Leftarrow$$

Subst. ② into ①:

$$V_1 = V_2 \frac{50}{1.25}\left(\frac{1}{200} - \frac{1}{j100}\right)$$

$$\therefore \frac{V_1}{V_2}\Big|_{I_1=0} = 0.2 + j0.4 = h_{12}$$

Notice that the Circuit is NOT reciprocal due to the dependent source therefore $h_{12} \neq -h_{21}$.

Finally, to obtain h_{22}:

$\frac{I_2}{V_2}\Big|_{I_1=0} \longrightarrow$ Port 1 open circuit

Same configuration as in the case of h_{12}

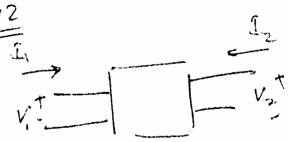
here $h_{22} = \frac{I_2}{V_2}\Big|_{I_1=0}$ which is directly equation (2) above (see $\Leftarrow$)

$$\therefore \frac{I_2}{V_2}\Big|_{I_1=0} = h_{22} = \frac{1}{1.25}\left(\frac{1}{200} + j\frac{1}{100}\right)$$

$$= (4 + j8) \times 10^{-3}$$

$$= (0.004 + j\,0.008).$$

18-11, 18-12



Port (2) open $I_2 = 0$

$V_1 = 20\times10^{-3}$

$V_2 = -5$

$I_1 = 0.25\times10^{-6}$

Port (2) sc → $V_2 = 0$

$I_1 = 200\times10^{-6}$

$I_2 = 50\times10^{-6}$

$V_1 = 10$

Find the g parameters and the Y-parameters

$I_1 = g_{11}V_1 + g_{12}I_2$ --- (1)

$V_2 = g_{21}V_1 + g_{22}I_2$ --- (2)

$g_{11} = \frac{I_1}{V_1}\Big|_{I_2=0}$ ∴ $g_{11} = \frac{25\times10^{-8}}{20\times10^{-3}} = 1.25\times10^{-5}$

$g_{21} = \frac{V_2}{V_1}\Big|_{I_2=0}$ ∴ $g_{21} = -\frac{5}{20}\times10^{3} = -250$

Next

as $V_2 = 0$ $\xrightarrow{\text{from (2)}}$ $g_{21}V_1 = -g_{22}I_2$

subst. in (1):

$\cancel{I_1 = g_{11}\cdot\frac{-g_{22}}{g_{21}}I_2 + g_{12}I_2}$

Subst. $g_{21} = -250$

$V_1 = 10$, $I_2 = 50\times10^{-6}$

∴ $-250\times10 = -g_{22}\times50\times10^{-6}$

∴ $g_{22} = \frac{2500}{50}\times10^{+6} = 50\times10^{6}$

Also from (1):

$I_1 = g_{11}V_1 + g_{12}I_2$ independent from V_2

∴ $200\times10^{-6} = 1.25\times10^{-5}\times10 + g_{12}\times50\times10^{-6}$

$75\times10^{-6} = g_{12}\times50\times10^{-6}$

$g_{12} = \frac{75}{50} = 1.50$

Y-parameters:

$I_1 = Y_{11}V_1 + Y_{12}V_2$ -- (3)

$I_2 = Y_{21}V_1 + Y_{22}V_2$ — (4)

$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} = \frac{200\times10^{-6}}{10} = 20\times10^{-6}$

$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0} = \frac{50\times10^{-6}}{10} = 5\times10^{-6}$

Next, regardless of I_2, equation (3) holds true all the time: from the condition $I_2 = 0$, subst. in (3):

$0.25\times10^{-6} = Y_{11}\times20\times10^{-3} + Y_{12}\times(-5)$

∴ $0.25\times10^{-6} = 20\times10^{-6}\times20\times10^{-3} + 5Y_{12}$

$0.15\times10^{-6} = 5Y_{12}$ ⇒ $Y_{12} = 0.03\times10^{-6}$

Finally from (4), at $I_2 = 0$

$Y_{21}V_1 = -Y_{22}V_2$

$5\times10^{-6}\times20\times10^{-3} = +Y_{22}\times5$

$Y_{22} = \frac{10^{-7}}{5} = 2\times10^{-8}$.

Check:

$Y_{12} = \frac{g_{12}}{g_{22}} = \frac{1.5}{50}\times10^{-6} = 3\times10^{-8}$

$Y_{21} = -\frac{g_{21}}{g_{22}} = -\frac{(-)250}{50}\times10^{-6} = 5\times10^{-6}$

$Y_{22} = \frac{1}{g_{22}} = \frac{1}{50}\times10^{-6} = 2\times10^{-8}$

$Y_{11} = \frac{g_{11}g_{22} - g_{12}g_{21}}{g_{22}} = \frac{625+375}{50\times10^{6}} = 20\times10^{-6}$