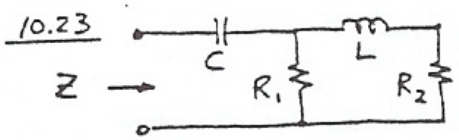
EE205: HW#4

10.23



$$Z = \frac{1}{j\omega C} + \frac{R_1(R_2+j\omega L)}{R_1+R_2+j\omega L} = \frac{R_1+R_2+j\omega L + j\omega R_1 C(R_2+j\omega L)}{j\omega C(R_1+R_2+j\omega L)}$$

$$= \frac{R_1+R_2-\omega^2 R_1 LC + j\omega(L+R_1R_2C)}{-\omega^2 LC + j\omega(R_1+R_2)C} \cdot \frac{-\omega^2 LC - j\omega(R_1+R_2)C}{-\omega^2 LC - j\omega(R_1+R_2)C}$$

$$Z = \frac{(R_1+R_2-\omega^2 R_1 LC)(-\omega^2 LC) + \omega^2(L+R_1R_2C)(R_1+R_2)C - j\omega^3 LC(L+R_1R_2C) - j\omega(R_1+R_2)C(R_1+R_2-\omega^2 R_1 LC)}{(-\omega^2 LC)^2 + [\omega(R_1+R_2)C]^2}$$

At resonance,

$$-j\omega^3 LC(L+R_1R_2C) - j\omega(R_1+R_2)C(R_1+R_2-\omega^2 R_1 LC) = 0$$

$$-\omega^2 L(L+R_1R_2C) - (R_1+R_2)^2 + (R_1+R_2)C\omega^2 R_1 L = 0$$

cont'd

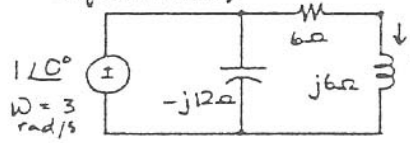
$$-L^2 - \omega^2 LR_1R_2C + R_1^2C\omega^2 L + R_1R_2LC\omega^2 = (R_1+R_2)^2$$

$$(R_1^2LC - L^2)\omega^2 = (R_1+R_2)^2$$

$$\omega^2 = \frac{(R_1+R_2)^2}{R_1^2LC - L^2} = \left(\frac{R_1+R_2}{R_1}\right)^2 \frac{1}{LC - \frac{L^2}{R_1^2}}$$

$$\therefore \omega_r = \frac{R_1+R_2}{R_1} \frac{1}{\sqrt{LC - \frac{L^2}{R_1^2}}}$$

10.26 From the text (Equation 10.6) $\omega_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = \sqrt{18 - \frac{36}{4}} = 3$ rad/s



$\underline{I} = \frac{1\angle 0^\circ}{6+j6} = \frac{1\angle 0^\circ}{6\sqrt{2}\angle 45^\circ}$

$= \frac{1}{6\sqrt{2}}\angle -45^\circ$

$w_c(t) = \frac{1}{2}Cv^2(t) = \frac{1}{2}\left(\frac{1}{36}\right)\cos^2 3t = \frac{1}{72}\cos^2 3t = \frac{1}{72}\left[\frac{1}{2}(1+\cos 6t)\right]$

$w_L(t) = \frac{1}{2}Li^2(t) = \frac{1}{2}(2)\left[\frac{1}{72}\cos^2(3t-45^\circ)\right] = \frac{1}{72}\cos^2(3t-45^\circ)$

$= \frac{1}{72}\left(\frac{1}{2}\right)\left[1+\cos(6t-90^\circ)\right]$

$w_c(t) + w_L(t) = \frac{1}{144}\left[1+\cos 6t + 1 + \sin 6t\right]$

$= \frac{1}{144}\left[2+\sqrt{2}\cos(6t-45^\circ)\right]$

$w_{max} = \frac{2+\sqrt{2}}{144}$ J

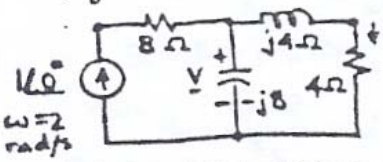
$P_R = \frac{1}{2}|\underline{I}|^2 R = \frac{1}{2}\left(\frac{1}{72}\right)6 = \frac{1}{24}$ W $\quad T = \frac{2\pi}{\omega} = \frac{2\pi}{3}$ s

$\rightarrow P_R T = \frac{2\pi}{72}$ J

$Q = 2\pi\left(\frac{w_{max}}{P_R T}\right) = 2\pi\left(\frac{\frac{2+\sqrt{2}}{144}}{\frac{2\pi}{72}}\right) = \frac{2+\sqrt{2}}{2} = 1 + \frac{1}{\sqrt{2}}$

$Q = 1.707$

10.27 From the text, (Equation 10.6) $\omega_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = \sqrt{8 - \frac{16}{4}} = 2$ rad/s.



$\underline{I} = \frac{-j8}{-j8+j4+4}(1\angle 0^\circ) = \frac{-j8}{4-j4}$

$\underline{I} = \frac{8\angle -90^\circ}{4\sqrt{2}\angle -45^\circ} = \sqrt{2}\angle -45^\circ$

$\underline{V} = (4+j4)\underline{I} = (4\sqrt{2}\angle 45^\circ)(\sqrt{2}\angle -45^\circ) = 8$

$w_C(t) = \frac{1}{2}Cv^2(t) = \frac{1}{2}\left(\frac{1}{16}\right)(64\cos^2 2t) = 2\cos^2 2t = 2\left[\frac{1}{2}(1+\cos 4t)\right]$

$w_L(t) = \frac{1}{2}Li^2(t) = \frac{1}{2}(2)\left[2\cos^2(2t-45^\circ)\right] = 2\left(\frac{1}{2}\right)\left[1+\cos(4t-90^\circ)\right]$

$w_C(t) + w_L(t) = 1+\cos 4t + 1 + \sin 4t = 2+\sqrt{2}\cos(4t-45^\circ)$

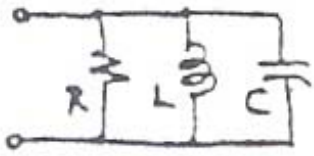
$\therefore w_m = 2+\sqrt{2}$ J

$P_4 = \frac{1}{2}|\underline{I}_4|^2(4) = \frac{1}{2}(2)(4) = 4 \qquad P_8 = \frac{1}{2}(1)^2 8 = 4$

$P_R = P_4 + P_8 = 8$ W $\quad T = \frac{2\pi}{\omega} = \frac{2\pi}{2}$ s $\Rightarrow \quad P_R T = 8\pi$ J

$Q = 2\pi \frac{w_m}{P_R T} = 2\pi \frac{2+\sqrt{2}}{8\pi} = \frac{2+\sqrt{2}}{4} = \underline{0.85}$

10.33



$R = \frac{50\times 10^{-3}}{50(0.005\times 10^{-6})} = 2\times 10^5 = 200\,\text{k}\Omega$

$L = 50$ mH $\qquad C = 0.005\,\mu$F

$Q = R\sqrt{\frac{C}{L}} = 2(10^5)\sqrt{\frac{0.005(10^{-6})}{50(10^{-3})}} = 2(10^5)\sqrt{10^{-7}} = \underline{632}$