King Fahd University of Petroleum & Minerals
Electrical Engineering Department
EE570: Stochastic Processes (122)
Quiz5: Random Processes (Frequency Domain & LTI System Response)
Dr. Ali Hussein Muqaibel

Name: KEY Serial Number ⊙

State whether or not the following function can be a valid power density spectrum (Justify your answer)

$\cos(4\omega)\, e^{-\omega^2+j2\omega}$

No it is not valid because it is not real. not nonnegative

An ideal low-pass filter with a mid-band amplitude gain of 8 and bandwidth of 4 rad/s has noise $X(t)$ at its input with power spectrum

$$S_{XX}(\omega) = \left(\frac{50}{\sqrt{8\pi}}\right) e^{-\frac{\omega^2}{8}}$$

What is the noise power at the filter's output?

$$f_X(x) = \frac{1}{\sqrt{(2\pi\sigma^2)}} e^{\frac{-(x-a_X)^2}{2\sigma_X^2}}$$

$$F_X(x) = \int_{-\infty}^{x} f_X(\xi)d\xi = F\left(\frac{x-a_X}{\sigma_X}\right)$$

Table is attached at the back

$S_{YY}(\omega) = S_{XX}(\omega)\,|H(\omega)|^2$

$P_{YY}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{XX}(\omega)\,|H(\omega)|^2\, d\omega$

$= \frac{1}{2\pi}\int_{-4}^{4} S_{XX}(\omega)\, 64\, d\omega$

$= \frac{64}{2\pi}\int_{-4}^{4} \left(\frac{50}{\sqrt{8\pi}}\right) e^{-\frac{\omega^2}{8}}\, d\omega$

$= \frac{(50)(64)}{2\pi}\int_{-4}^{4} \frac{1}{\sqrt{2\pi(4)}}\, e^{-\frac{\omega^2}{2(4)}}\, d\omega = \frac{(25)(64)}{\pi}\left(F\left(\frac{4-0}{2}\right) - F\left(\frac{-4-0}{2}\right)\right)$

$= \frac{(25)(64)}{\pi}\left(F(2) - F(-2)\right) = \frac{(25)(64)}{\pi}\left(2F(2) - 1\right) = 486.17$

($F(2) \approx 0.9773$)

$a_X = 0$

$\sigma_X^2 = 4$

48617

Good Luck, **Dr. Ali Muqaibel**