King Fahd University of Petroleum & Minerals
Electrical Engineering Department
EE570: Stochastic Processes (122)
Quiz 4: Random Processes (Temporal Domain)
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For two zero-mean, jointly w.s.s. random processes $X(t)$ and $Y(t)$, it is known that $\sigma_X^2 = 4$ and $\sigma_Y^2 = 9$. Explain why each of the following functions **cannot** apply to the processes if they have no periodic components.

(a) $R_{YY}(\tau) = 3u(\tau)\exp(-\tau)$ Not even symmetric or $R_{YY}(0) \neq \sigma_Y^2 = 9$ 1

(b) $R_{XY}(\tau) = 9(1+2\tau^2)^{-1}$ $R_{XY}(0) = 9 \nless \sqrt{(4)(9)}$ 1
Bound is not satisfied $R_{XY}(0) \le \sqrt{R_{XX}(0) R_{YY}(0)}$

(c) $R_{XX}(\tau) = 5\left[\frac{\sin(3\tau)}{3\tau}\right]^2$ $R_{XX}(0) = 5 \neq 4$
" does " satisfy. $\lim_{\tau\to\infty} R_{XX}(\tau) = 0$ It should Equal to 4 1

Aircraft arrive at an airport according to a Poisson process at a rate of 6 per hour. All aircrafts are handled by one air-traffic controller. If the controller takes 5-mintues coffee break, what is the probability that he will miss one or more arriving aircrafts?

$P[N(t)=k] = \frac{(\lambda t)^k}{k!} e^{-\lambda t}$ $\lambda = 6 \frac{\text{aircrafts}}{\text{Hour}} = 0.1 /\text{min}.$ 1

$P[N(t)=0] = \frac{(0.1(5))^0}{0!} e^{-0.1(5)} = e^{-0.5} \Rightarrow P[1 \text{ or more}] = 1 - e^{-0.5} = 0.3935$ 1

At the receiver of an AM radio, the received signal contains a cosine carrier signal at the carrier frequency f_c with a random phase Θ that is a sample value of the uniform $(0,\pi)$ random variable. The received carrier signal is

$$X(t) = A\cos(2\pi f_c t + \Theta)$$

What are the expected value and autocorrelation of the process $X(t)$? Is the process stationary in any sense?

$E[X(t)] = E[A\cos(2\pi f_c t + \theta)] = A\,E[\cos(2\pi f_c t + \theta)]$

$= A\int_0^\pi \cos(2\pi f_c t + \theta)\frac{1}{\pi}\,d\theta = \frac{A}{\pi} \sin(2\pi f_c t + \theta)\Big|_0^\pi$ 2

$= \frac{A}{\pi}[\sin(2\pi f_c t + \pi) - \sin(2\pi f_c t)] = \frac{-2A}{\pi}\sin(2\pi f_c t)$

$E[X(t)X(t+\tau)] = A^2 E[\cos(2\pi f_c t + 2\pi f_c \tau + \theta)\cos(2\pi f_c t + \theta)]$

$= \frac{A^2}{2} E[\cos(2\pi f_c \tau) + \cos(4\pi f_c t + 2\pi f_c \tau + 2\theta)]$ 2θ $0 \to 2\pi$

2 $= \frac{A^2}{2}\cos(2\pi f_c \tau)$ 0

Good Luck, **Dr. Ali Muqaibel**

It is not stationary because
1 mean is not constant.