

King Fahd University of Petroleum & Minerals

Electrical Engineering Department

EE570: Stochastic Processes (122)

Quiz 3: Random Variables and Expectations

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A random variable has a probability density

$$f_X(x) = \begin{cases} a(1-x^4) & 0 < x \leq 1 \\ 0 & \text{elsewhere in } x \end{cases}$$

Find the value of the constant  $a$ ?

Area under pdf = 1

$$\int_0^1 a(1-x^4) dx = a \left[ x - \frac{x^5}{5} \right]_0^1$$

$$= a \left[ 1 - \frac{1}{5} \right] = \frac{4}{5} a = 1$$

$$\Rightarrow a = \frac{5}{4} = 1.25$$

Find the variance of the random variable  $X$ ?

$$\text{mean} = a \int_0^1 x(1-x^4) dx = a \int_0^1 (x - x^5) dx = a \left[ \frac{x^2}{2} - \frac{x^6}{6} \right]_0^1$$

$$= a \left[ \frac{1}{2} - \frac{1}{6} \right] = a \frac{1}{3} = \frac{5}{4} \cdot \frac{1}{3} = \frac{5}{12}$$

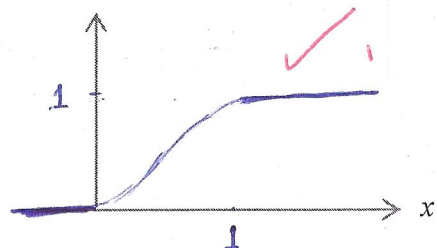
$$E[X^2] = a \int_0^1 x^2(1-x^4) dx = a \int_0^1 (x^2 - x^6) dx = a \left[ \frac{x^3}{3} - \frac{x^7}{7} \right]_0^1$$

$$= a \left[ \frac{1}{3} - \frac{1}{7} \right] = a \left[ \frac{4}{21} \right] = \frac{5}{4} \cdot \frac{4}{21} = \frac{5}{21}$$

$$\sigma_x^2 = E[X^2] - (E[X])^2 = \frac{5}{21} - \left( \frac{25}{144} \right) = 0.0645$$

Find and sketch the CDF,  $F_X(x)$

$$F_X(x) = \begin{cases} \int_0^x a(1-x^4) dx = \frac{5}{4} \left[ x - \frac{x^5}{5} \right] & 0 < x \leq 1 \\ 0 & x < 0 \\ 1 & x > 1 \end{cases}$$



Find the probability that  $x < 0.25$  given that  $x < 0.5$ .

$$F_X(x < 0.25 | x < 0.5) = \frac{F_X(0.25)}{F_X(0.5)} = \frac{\frac{1}{4} - \frac{1}{4^5 \cdot 5}}{\frac{1}{2} - \frac{1}{2^5 \cdot 5}} = \frac{0.2498}{0.49375} = 0.506$$