EE 570 Stochastic Processes
HW#7 Queuing Theory
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Problem #1

Patient and impatient customers. Consider an $M/M/r$ queue with arrival rate λ and service rate μ that contains both patient and impatient customers at its point. If all servers are busy, patient customers join the queue and wait for service, while impatient customers leave the system instantly. If p represents the probability of an arriving customer to be patient, show that when $\frac{p\lambda}{r\mu} < 1$ the steady state distribution in the system is given by

$$p_n = \begin{cases} \dfrac{(\lambda/\mu)^n}{n!} p_0 & n < r \\ \dfrac{\left(\frac{\lambda}{\mu}\right)^r}{r!} \left(\dfrac{p\lambda}{r\mu}\right)^{n-r} p_0 & n \geq r \end{cases}$$

where

$$p_0 = \frac{1}{\sum_{n=0}^{r-1} \frac{(\lambda\mu)^n}{n!} + \frac{(\lambda/\mu)^r}{r!\,(1-\frac{p\lambda}{r\mu})}}$$

(Hospital, restaurants, barber shops, department stores, and telephone exchanges all lose customers who are either inherently impatient or cannot afford to wait.)

Problem#2

Describe the following queuing systems $M/M/1$, $M/D/1/K$, $M/G/3$, $D/M/2$, $G/D/1$, $D/D/2$.

Problem#3

A data communication line delivers a block of information every 10 microsecond. A decoder checks each block for the errors and corrects the errors if necessary. It takes 1 μs to determine whether a block has any error. If the block has one error, it takes 5 μs to correct it, and if it has more than one error it takes 20 μs to correct the error. Blocks wait in a queue when the decoder fails behind. Suppose that the decoder is initially empty and that the numbers of errors in the first ten blocks are 0,1,3,1,0,4,0,1,0,0.

a) Plot the number of blocks in the decoder as a function of time.
b) Find the mean number of blocks in the decoder.
c) What percentage of the time is the decoder empty?

Problem#4

A very popular barbershop is always full. The shop has two barbers and three chairs for waiting, and as soon as a customer completes his service and leaves the shop, another enters the shop. Assume the mean service time is m.

a) Use little's formula to relate the arrival rate and the mean time spent in the shop.
b) Use little's formula to relate the arrival rate and the mean time spent in the service.
c) Use above formulas to find an expression for the mean time spent in the system in terms of the mean service time.

Problem#5

In Problem#3, suppose that the probabilities of zero, one, and more than one errors are p_0, p_1, and p_2 respectively. Use little's formula to find the mean number of blocks in the decoder.

Problem#6

a) Find $P[N \geq n]$ for an M/M/1 system.
b) What is the maximum allowable arrival rate in a system with service rate μ, if we require that $P[N \geq 10] = 10^{-3}$.

Problem#7

Consider M/M/1 queuing system with arrival rate λ customers/second.

a) Find the service rate required so that the average queue is five customers (i.e., $E[N_q = 5]$).
b) Find the service rate required so that the queue that forms from time to time has mean 5 (i.e., $E\left[N_q \middle| N_q > 0 = 5\right]$).
c) Which of the two criteria, $E[N_q]$ or $E\left[N_q \middle| N_q > 0\right]$, do you consider the more appropriate?

Problem#8

Consider M/M/1/2 queuing system in which each customer accepted into a system brings in a profit of \$5 and each customer rejected results in a loss of \$1. Find the arrival rate at which the system breaks even.

Problem#9

Customers arrive at a shop according to a Poisson process of rate 12 customers per hour. The shop has two clerks to attend to the customers. Suppose that it takes a clerk an exponentially distributed amount of time with mean 5 minutes to service one customer.

a) What is the probability that an arriving customer must wait to be served?
b) Find the mean number of customers in the system and the mean time spent in the system.
c) Find the probability that there are more than 4 customers in the system.

Problem#10

A tool rental shop has four floor sanders. Customers for the floor sanders arrive according to a Poisson process at a rate of one customer every two days. The average rental time is exponentially distributed with mean two days. If the shop has no floor sanders available, the customers go to the shop across the street.

a) Find the proportion of customers that go to the shop across the street.
b) What is the mean number of floor sanders rented out?
c) What is the increase in lost customers if one of the sanders breaks down and isn't replaced?

It is the mark of an educated mind to be able to entertain a thought without accepting it.
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Good Luck, Dr. Muqaibel