EE 570 Stochastic Processes
HW#6 Markov Chains
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Problem #1

Let M_n denote the sequence of sample means from an iid random process X_n:

$$M_n = \frac{X_1 + X_2 + \cdots + X_n}{n}$$

Show that M_n is a Markov process by representing M_n in terms of current input and previous output.

Problem#2

One of the following statements is true. Indicate and support your decision

a) A Markov process has independent increments.
b) A process which has independent increment property is a Markov process.

Problem#3

An urn initially contains five black balls and five white balls. The following experiment is repeated indefinitely: A ball is drawn from urn; if the ball is white it is put back in the urn, otherwise it is left out. Let X_n be the number of black balls remaining in the urn after n draws from the urn.

a) Is X_n a Markov process? If so, find the appropriate transition probabilities.
b) Do the transition probabilities depend on n?

Problem#4

Let X_n be the Markov chain defined in Problem#3

a) Find the one-step transition probability matrix P for X_n.
b) Find the two-step transition probability matrix P^2 by matrix multiplication. Check your answer by computing $p_{54}(2)$ and comparing it to the corresponding entry in P^2.
c) What happens to X_n as n approaches infinity? Use your answer to guess the limit of P^n as $n \to \infty$.

Problem#5

A machine consists of two parts that fail and are repaired independently. A working part fails during any given day with probability a. A part that is not working is repaired by the next day with probability b. Let $\mathrm{X_n}$ be the number of working parts in day n.

a) show that X_n is a three –state Markov chain and give its one-step transition probability matrix P.
b) show that the steady state pmf $\boldsymbol{\pi}$ is binominal with parameter $p = b/(a+b)$.
c) what do you expect is steady state pmf for a machine that consists of n parts?

Problem#6

A shop has n machine and one technician to repair them. A machine remains in the working state for an exponentially distributed time with parameter μ. The technician works on one machine at a time, and it takes him an exponentially distributed time of rate α to repair each machine. Let $X(t)$ be the number of working machines at time t.

a) Show that if $X(t) = k$, then the time until the next machine breakdown is an exponentially distributed random variable with rate $k\mu$.
b) Find the transition rate matrix $[\gamma_{ij}]$ and sketch the transition rate diagram for $X(t)$.
c) Write the global balance equations and find the steady state probabilities of $X(t)$.

Problem#7

Consider the single-server queuing system. Suppose that at most K customers can be in the system at any time. Let $N(t)$ be the number of customers in a time t. Find the steady state probabilities for $N(t)$.

It is possible to store the mind with a million facts and still be entirely uneducated.
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Good Luck, Dr. Muqaibel