KFUPM-EE DEPT.
EE570- Stochastic Processes
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Assignment # 4 | Due: Week 10, Class 1

Random Processes

Version 2.0 revised

Note: for this specific assignment a group of two students can work together and submit one solution.

1. Sample functions in a discrete random process are constants; that is

$$X(t) = C = constant$$

where C is a discrete random variable having possible values $c_1 = 1, c_2 = 2, and\ c_3 = 3$ occurring with probabilities 0.7, 0.2, and 0.1, respectively.

   a) Is $X(t)$ deterministic?
   b) Find the first order density function of $X(t)$ at any time t.

2. Assume that an ergodic process $X(t)$ has an autocorrelation function

$$R_{XX}(\tau) = 20 + \frac{2}{6+\tau^2}[1 + 3\cos(6\tau)]$$

   a) Find $|\bar{X}|$.
   b) Does this process have a periodic component?
   c) What is the average power in $X(t)$?

3. A random process $Y(t) = X(t) - X(t+\tau)$ is defined in terms of a process $X(t)$ that is at least wide-sense stationary.
   a) Show that the mean value of $Y(t)$ is 0 even if $X(t)$ has a nonzero mean value.
   b) Show that: $\sigma_Y^2 = 2[R_{XX}(0) - R_{XX}(\tau)]$
   c) If $Y(t) = X(t) + X(t+\tau)$, find $E[Y(t)]$, and σ_Y^2. How do these results compare to those of part (a) and (b).

4. Given two random processes $X(t)$ and $Y(t)$. Find the expressions for autocorrelation function of $W(t) = X(t) + Y(t)$ if :
   a) $X(t) and\ Y(t)$ are correlated.
   b) They are uncorrelated.
   c) They are uncorrelated with zero means.

5. Determine the largest constant K such that the function

$$R_{XY}(\tau) = Ke^{-\tau^2}\sin(\pi\tau)$$

is a valid corss-correlation function of two jointly wide-sense stationary processes $X(t)$ and $Y(t)$ for which $E[X^2(t)] = 6$ and $E[Y^2(t)] = 9$.

6. A small store has two check-out lanes that develop waiting lines. If more than two costumers arrive in any three minute interval. Assume that a Poisson

process describes the number of customers that arrive for check-out. Find the probability of a waiting line if the average rate of customer arrival is

a) 4 per minute
b) 2 per minute
c) 0.5 per minute

7. Let $X(t)$ and $Y(t)$ be independent Gaussian random process with zero means and the same covariance function $C_{XX}(t_1, t_2)$. Define the following amplitude modulated signal $Z(t) = X(t)\cos\omega t + Y(t)\sin\omega t$
   a) Find the mean and autocovariance of $Z(t)$
   b) Find the pdf of Z(t)
   c) Support your answer with Matlab *Monte Carlo* Simulation. Make all needed assumptions and values

8. Let M_n be the discrete-time process defined as the sequence of sample means of an iid sequence:

$$M_n = \frac{X_1 + X_2 + \cdots + X_n}{n}$$

   a) Find the mean, variance, and covariance of M_n.
   b) Does M_n have independent increments? Stationary increments?

9. Let X_n consist of an iid sequence of Poisson random variables with mean α.

   a) Find the pmf of the sum process S_n .
   b) Find the joint pmf of S_n and S_{n+k}.

10. Noise impulses occur on a telephone line according to Poisson process of rate λ.

   a. Find the probability that no impulses occur during the transmission of a message that is t seconds long.
   b. Suppose that the message is encoded so that the errors caused by a single impulse can be corrected. What is the probability that the *t*-seconds message is either error-free or correctable?
   c. Support your answer with Matlab *Mote Carlo* simulation. Assume values for λ and t.

In order to learn Random Processes
"*Take chances, make mistakes, get messy!*" **in the HW assignment!**

Good luck , **Dr. Ali H. Muqaibel**