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Design of Wide-Band Metamaterials Using Multi-Element Split Ring Resonator Structures

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***Abstract*-The highly resonant characteristics of the split ring resonator can be significantly improved by introducing new pole pairs to Lorentz atomic model. This is equivalent to adding more elements to the basic cell. For wide-band metamaterials, the properties of the multi-element cell are carefully designed such that they correspond to the required resonance frequencies and damping terms. A new split ring resonator cell of metamaterials has been designed and simulated using the FDTD method. The simulation results were validated against analytical solutions.**

***Index Terms*- Metamaterials, auxiliary differential equation, FDTD, split ring resonator, Z-transform.**

I. INTRODUCTION

Metamaterials are traditionally designed using mainly two structures; a dense array of thin metallic wires for obtaining negative permittivity and an array of split-ring resonators (SRRs) for obtaining negative permeability. For the case of thin wires, Drude model is applicable and it has been shown [1] that this model exhibits a high-pass behavior for an incoming plane wave with the electric field parallel to the wires. SRRs have been proposed [2] for obtaining negative values for the permeability at certain frequencies near the resonance. They can be thought of as a small LC circuit that provides the required phase delay. This delay exceeds 90 degrees beyond the resonant frequency and thus the real part of the response becomes negative. The effective permeability of this material is approximated by the Lorentz model, which offers negative permeability for a limited frequency range and is also very sensitive to the change in frequency. Many variations of rings and rods have since been devised to achieve negative permittivity and permeability; including the edge-coupled SRR [3], the broadside SRR [4], the axially symmetric SRR [5], the omega SRR [6] and the S-ring type [7]. In this paper, a new methodology for improving the resonant characteristics of the split ring resonator is presented where new pole pairs are added to the basic SRR model. This is equivalent to adding more elements to the basic SRR cell. Each new pole pair has specific characteristics dictated by the pole position in the s-plane. These characteristics are then related to the physical dimensions of the SRRs. Matching the new multi-element metamaterial cell to free space over a wide frequency band is attempted. FDTD simulations have been performed and compared to analytical results.

II. POLE PLACEMENT AND CELL DESIGN

The effective permeability of the basic SRR proposed in [2] is approximated by the Lorentz model which is generally given by

$$\mu_{eff(\omega)} = \mu_o \left(1 + \frac{\omega_p^2}{\omega_o^2 - \omega^2 + j\omega\gamma}\right) \qquad (1)$$

where ω_p is the plasma frequency, ω_o is the natural resonant frequency and γ is the damping coefficient. Figure 1 shows the frequency response of this model, which exhibits a very narrow band of negative permeability values and is very sensitive to frequency changes. A lattice based on a multi-element SRR cell, similar to the one shown in figure 2, is proposed. If the cell is composed of N different SRRs (N poles), then, following [2], the relative effective permeability of the medium is given by

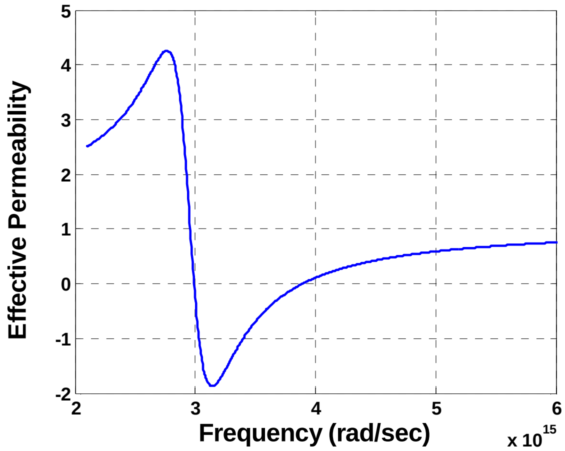

Fig.1. Frequency response of a single pole pair SRR.

$$\mu_{eff(\omega)} = 1 - \sum_{k=1}^{N} \frac{F_k}{1 + \frac{j2\sigma_k}{\omega r_k \mu_o} - \frac{3}{\pi^2 \mu_o \omega^2 C r_k^3}} \quad (2)$$

as a first-order approximation. In equation 2, $F_k = \pi r_k^2 / a^2$ is the fractional volume of the cell occupied by the interior of SRR k, a is the cell dimension, $C = \varepsilon_o / d$ is the capacitance per unit area between the two SRR sheets, r_k is the radius of the outer ring, σ_k is the surface resistivity of the SRR and d is the distance between the SRR rings.

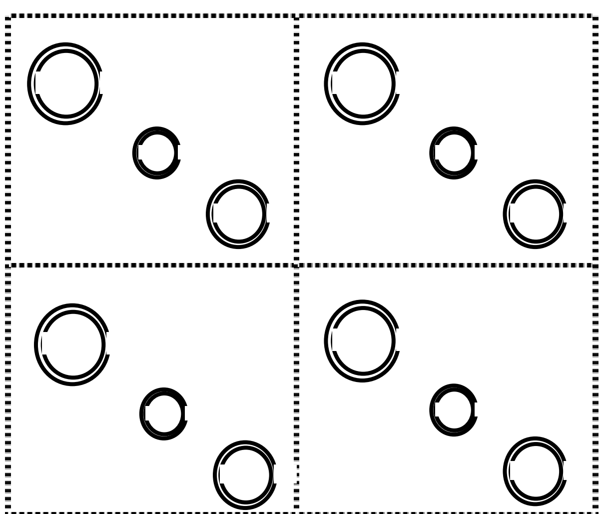

Fig.2. The proposed multi-element lattice with three SRRs per cell. Each SRR contributes a Lorentz pole pair to the effective permeability of the medium.

The correspondence between the pole locations and the dimensions of the SRRs is obtained as follows. The real and imaginary parts of pole k are, respectively, given by

$$\alpha_k = -\frac{\gamma_k}{2} \quad (3)$$

and

$$\beta_k^2 = \omega_{ok}^2 - \alpha_k^2 \quad (4)$$

The damping coefficient γ_k and the resonance frequency ω_{ok} are related to the physical dimensions of the SRR, respectively, by [2]

$$\gamma_k = \frac{2\sigma_k}{r_k \mu_o} \quad (5)$$

and

$$\omega_{ok} = \sqrt{\frac{3dc^2}{\pi^2 r_k^3}} \quad (6)$$

For example, for a 3-element cell design that produces a negative permeability of -1 over a wide frequency band (free-space matching), the s-plane pole pairs locations for a Lorentz approximation of the SRRs are shown in figure 3. The required resonance frequencies and damping terms can be calculated according to the required band over which constant negative permeability is needed. It should be mentioned that the presence of the additional zeros in expression 2 makes it more difficult to obtain a flat response as compared to the Lorentz model. For this particular 3-element cell example, the pole locations and the corresponding resonant frequencies are given in table 1. The resulting frequency response of the medium is shown in figure 4. It is evident that the more pole-pairs are introduced the more control is achieved over the frequency response.

III. FDTD ALGORITHM AND NUMERICAL RESULTS

For multi-term dispersion, inclusion of the material characteristics in the FDTD becomes complicated. Several techniques for the accommodation of the frequency-dependent dispersion relations into the time-domain FDTD algorithm have been reported in literature, including the auxiliary differential equation (ADE) method [8], the recursive convolution method [9] and Z-transform method [10]. The ADE-based solution for multi-term dispersion involves higher order derivatives and requires the use of matrix inversion. For three complex pole pairs of the Lorentz model for example, derivatives of the sixth order result. The update equations in this case contain several future quantities that are yet to be evaluated. So, the problem is transformed into a mixed explicit-implicit scheme that can be solved by matrix inversion. In this work, the procedure described in [11] that utilizes the Z-transform technique will be used.

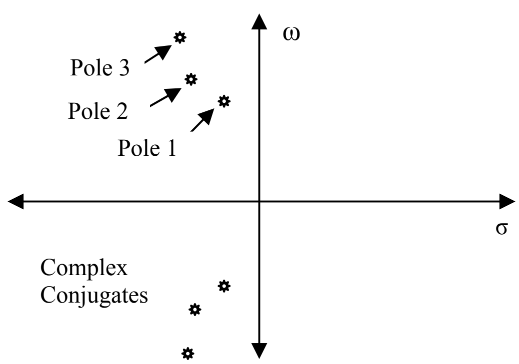

Fig.3. Pole placement in the s-plane.

Table 1: Lorentz model pole locations for flat response.

k	α x 10^{14}	β x 10^{15}	ω_o x 10^{15}	ω_p x 10^{15}
1	-1.8661	2.952	2.957	2.597
2	-6.6198	3.1921	3.26	2.9829
3	-9.4105	3.6457	3.765	2.7109

First let us consider a Lorentz media with N number of pole pairs. The magnetic flux density is related to the field intensity by the relation

$$B(\omega) = \mu_o H(\omega) + \sum_{k=1}^{N} \frac{\mu_o \omega_{pk}^2 H(\omega)}{\omega_{ok}^2 - \omega^2 + j\omega\tau} \quad (7)$$

Let us define

$$T_k(\omega) = \frac{\mu_o \omega_{pk}^2 H(\omega)}{\omega_{ok}^2 - \omega^2 + j\omega\tau} = \frac{\mu_o \omega_{pk}^2 H(\omega)}{s^2 + s\tau + \omega_{ok}^2} \quad (8)$$

where we have replaced $j\omega$ with $s = \sigma + j\omega$ to transform into the s-domain. Equation 8 represents a very common second order system and its response in the time domain is given by

$$T_k(t) = \frac{\mu_o \omega_{pk}^2}{\beta} e^{\alpha t} \sin(\beta t) * H(t) \quad (9)$$

For a discrete system, we can write the above equation as

$$T_k(n\,\Delta t) = \frac{\mu_o \omega_{pk}^2}{\beta} e^{\alpha\, n\Delta t} \sin(\beta\, n\Delta t) * H(n\Delta t) \quad (10)$$

Utilizing the following Z-transform pair

$$Z\{e^{\alpha\, n\Delta t} \sin(\beta\, n\Delta t)\} = \frac{Z^{-1} e^{\alpha\Delta t} \sin(\beta\Delta t)}{1 - 2Z^{-1} e^{\alpha\Delta t} \cos(\beta\Delta t) + Z^{-2} e^{2\alpha\Delta t}} \quad (11)$$

we obtain the expression for magnetic flux density as

$$B(Z) = \mu_o H(Z) + \sum_{k=1}^{N} \frac{Z^{-1} \frac{\mu_o \omega_{pk}^2}{\beta} e^{\alpha\Delta t} \sin(\beta\Delta t)}{1 - 2Z^{-1} e^{\alpha\Delta t} \cos(\beta\Delta t) + Z^{-2} e^{2\alpha\Delta t}} H(Z)\Delta t \quad (12)$$

Note that the Δt factor in equation 12 is due to the convolution in equation 10. Let us define a new variable $S_k(Z)$ as

$$S_k(Z) = \frac{\frac{\mu_o \omega_{pk}^2}{\beta} e^{\alpha\Delta t} \sin(\beta\Delta t)}{1 - 2Z^{-1} e^{\alpha\Delta t} \cos(\beta\Delta t) + Z^{-2} e^{2\alpha\Delta t}} H(Z)\Delta t \quad (13)$$

Taking the inverse Z-transform we obtain

$$S_k^n = 2e^{\alpha\Delta t} \cos(\beta\Delta t)\, S_k^{n-1} - e^{2\alpha\Delta t} S_k^{n-2} + \frac{u_o \omega_{pk}^2}{\beta_k} e^{\alpha\Delta t} \sin(\beta\Delta t)\, H^n \Delta t \quad (14)$$

Equation 14 is applicable to a general Lorentz media in which there are no zeros at the origin of the s-plane. However, as two additional zeros at the origin are present, as indicated by equation 2, equation 14 needs to be modified as

$$S_k^n = 2e^{\alpha\Delta t}\cos(\beta\Delta t)\,S_k^{n-1} - e^{2\alpha\Delta t}S_k^{n-2} - \frac{\mu_o\omega_{pk}^2}{\beta_k}e^{\alpha\Delta t}(\sin(\beta\Delta t)\cos(\varphi) - \cos(\beta\Delta t)\sin(\varphi))H^n\Delta t \quad (15)$$

where

$$\varphi = \tan^{-1}\left(-\frac{2\alpha\beta}{\alpha^2-\beta^2}\right) \quad (16)$$

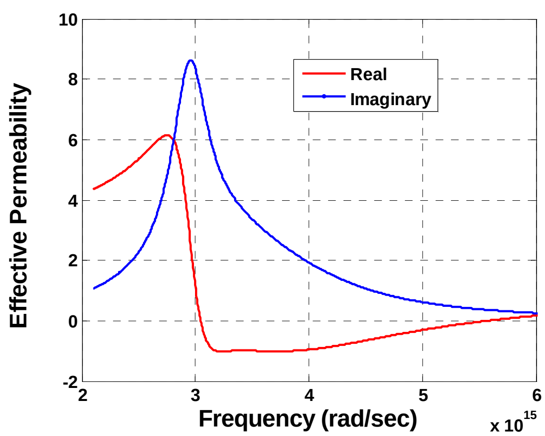

Fig.4. Frequency response due to combined effect of the three different SRRs.

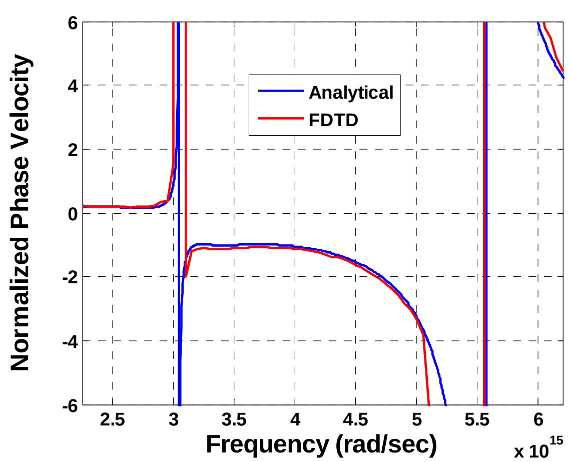

Fig.5. Simulated normalized phase velocity.

In deriving equation 15 we have used the fact that $\alpha_k << \beta_k$. Note that, using equation 15, the update equation for *H*-field is given by

$$H^n = \frac{B^n - \sum_{k=1}^{N} S_k^{n-1}}{u_o} \quad (17)$$

where B^n is calculated using the finite difference expression of Maxwell's curl equation and S_k^{n-1} is the value calculated in the previous iteration. Similar expressions for the *D* and *E* fields can be derived. For simplicity both permittivity and permeability are modeled by the same multi-pole dispersive model.

To test the solution algorithm, a monochromatic wave was incident from air onto the structure composed of 3-element SRR cells as given by table 1. For the sake of illustration, the 2D transverse magnetic (TM) polarization is considered. In order to calculate the phase velocity, the zero crossing of the wave inside the structure at two different points along the direction of propagation is observed. The distance between these two points was fixed, and by noting the time the wave takes to travel between these points, the phase velocity is calculated. This procedure was repeated at different input signal frequencies. The simulated phase velocity curve is shown in figure 5. An excellent agreement is obtained with the analytical values given by equation 1. Within the desired frequency band, the phase velocity is equal to the negative of free space velocity *c*. It is also interesting to note that the permeability remains less than one for a very large band which indicates the possibility of designing a very broad band material lighter than air. Therefore this material can be further explored for the development of optical cloaking device requiring such properties as remarked in [12]. These results are also supported by the reflection curve of figure 6. The curve is produced by launching a wideband optical pulse from air onto a material composed of the multi-element SRR structure considered in this example. Compared to the standard SRR, the figure shows the enhanced response of the multi-element design.

VI. CONCLUSION

A multi-pole dispersive model based on the split ring resonators has been proposed for the development of a broad band metamaterial at the optical frequencies. The proposed structure was simulated using the FDTD Z-transform method. It has been shown that such methodology offers much better results regarding the frequency response of the metamaterial as compared to that of a single SRR design. It is worth noting, however, that some means of overcoming the material loss must be investigated.

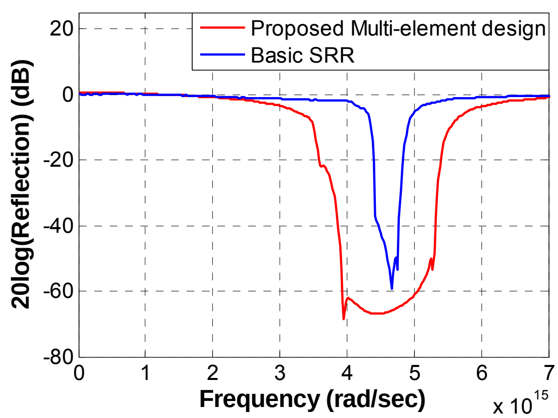

Fig.6. Reflection at the air interface with material composed of the the 3-element SRR cell.

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