Design of Wide-Band Metamaterials Based on the Split Ring Resonator Model

A. Ahmed and M. A. Alsunaidi
King Fahd University of Petroleum & Minerals, Saudi Arabia
msunaidi@kfupm.edu.sa

Abstract- It is shown that the highly resonant characteristic of split ring resonator (SRR) can be significantly improved by introducing new pole pairs to Lorentz atomic model, and by adjusting their resonant frequencies and damping terms. A new SRR cell of metamaterials containing all these required poles is then designed for the desired bandwidth.

1. INTRODUCTION

Metamaterials are traditionally designed using mainly two structures, a dense array of thin metallic wires (for obtaining negative permittivity) and an array of split-ring resonators (for obtaining negative permeability). For the case of thin wires, Drude model is applicable and it has been shown [1] that this model exhibits a high-pass behavior for an incoming plane wave with electric field parallel to the wires. SRRs have been proposed [2] for obtaining negative values for the permeability at certain frequencies near the resonance, SRR can be thought of as a small LC circuit that provides the required phase delay so the local fields will be almost out of phase with the incident field. The effective permeability of this material is approximated by the Lorentz model and is generally given by

$$u_{eff(\omega)} = u_o\left(1+\frac{\omega_p^2}{\omega_o^2-\omega^2+i\omega\gamma}\right) \qquad (1)$$

where ω_p is the plasma frequency, ω_o is the natural resonant frequency and γ is the damping coefficient. Figure 1 shows the frequency response of this model, the response shows a very narrow band of negative permeability values and is very sensitive to frequency changes.

In this paper, we introduce new pole pairs to equation 1 in order to increase the bandwidth of interest and to match the metamaterial to free space over a much wider band. Each new pole pair needs to have different characteristics dictated by the pole position in the s-plane; these characteristics are then related to the physical dimensions of the SRRs.

2. POLE PLACEMENT AND CELL DESIGN

The SRR proposed in [2] is considered and it will be shown that by using three different SRRs, the frequency response can be greatly improved. The lattice cell comprises of three SRRs. The effective permeability of the medium for this structure is given by

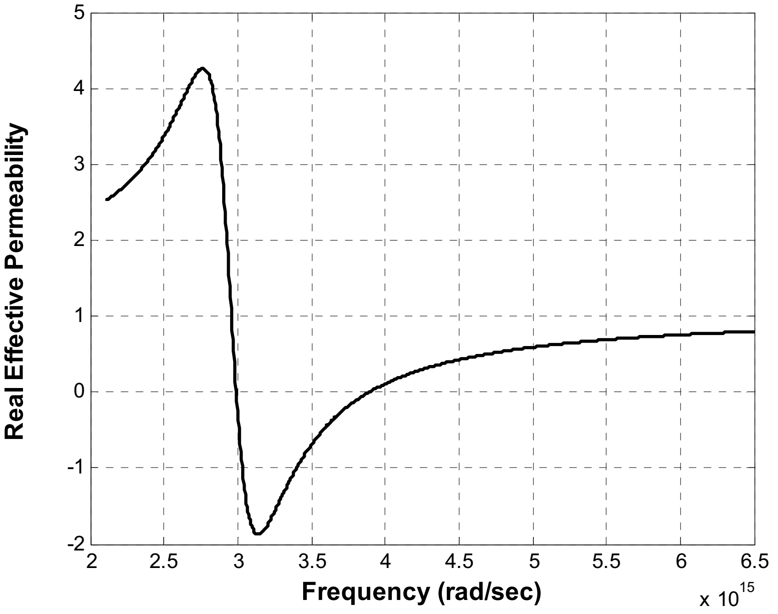

Figure 1. Frequency response of a single pole pair SRR.

$$u_{eff(\omega)} = 1 - \sum_{k=1}^{N} \frac{F_k}{1 + \frac{i2\sigma_k}{\omega r_k u_o} - \frac{3}{\pi^2 u_o \omega^2 C r_k^3}} \qquad (2)$$

$$F = \frac{\pi r^2}{a^2} \qquad (3a)$$

$$C = \frac{\varepsilon_o}{d} \qquad (3b)$$

where F_k is the fractional volume of the cell occupied by the interior of the cylinder, C is the capacitance per unit area between the two sheets, r_k is the radius of the outer ring and N is the number of SRRs per unit cell. Figure 2 shows the frequency response of this structure. The s-plane pole locations for generating the desired frequency response is shown in figure 3. These pole locations dictate the dimensions of the SRRs as

$$\alpha_k = -\frac{\gamma_k}{2} \qquad (4a) \qquad and \qquad \omega_{ok} = \sqrt{\frac{\gamma_k^2}{4} + \beta_k^2} \qquad (4b)$$

where α_k and β_k are the real part and the imaginary part of pole k, respectively. The damping coefficient γ_k and resonant frequency ω_{ok} are related to the physical dimensions of the SRR by

$$\gamma_k = \frac{2\sigma_k}{r_k u_o} \qquad (5a) \qquad and \qquad \omega_{ok} = \sqrt{\frac{3dc_o^2}{\pi^2 r_k^3}} \qquad (5b)$$

where σ_k is the surface resistivity of SRR and d is the distance between the SRRs. The required resonant frequencies and damping terms can be readily read from figure 3. These quantities are given at table 1.

Table 1

k	α	β	ω_o	γ	r	σ_k	d
1	-9.4104E+14	3.645E+15	3.765E+15	1.881E+15	6.0E-09	7.71E+00	1.321E-10
2	-6.6198E+14	3.191E+15	3.260E+15	1.323E+15	7.2E-09	6.51E+00	1.712E-10
3	-1.8661E+14	2.951E+15	2.957E+15	3.732E+14	8.5E-09	2.17E+00	2.319E-10

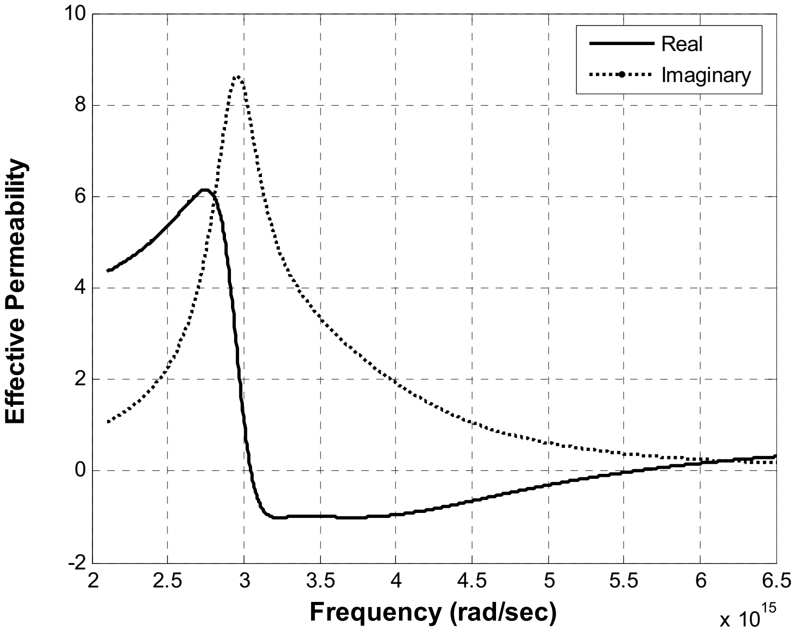

Figure 2. Frequency response due to combined effect of the three different SRRs.

3. FDTD FORMULATION

For simplicity only the 2D transverse magnetic (TM) polarization is considered. The multi-pole dispersive model can be simulated in a number of ways, including Recursive Convolution, PLRC and Z-Transform. In this paper, the Z-Transform approach has been used. The *H* field update equation is given by

$$H^n = \frac{B^n - \sum_{k=1}^{N} S_k^{n-1}}{u_o} \qquad (6)$$

with

$$S_k^n = 2e^{\alpha\Delta t}\cos(\beta\Delta t)\,S_k^{n-1} - e^{2\alpha\Delta t}S_k^{n-2} + \frac{u_o\omega_{pk}^2}{\beta_k}e^{\alpha\Delta t}\sin\,(\beta\Delta t)H^n\Delta t \qquad (7)$$

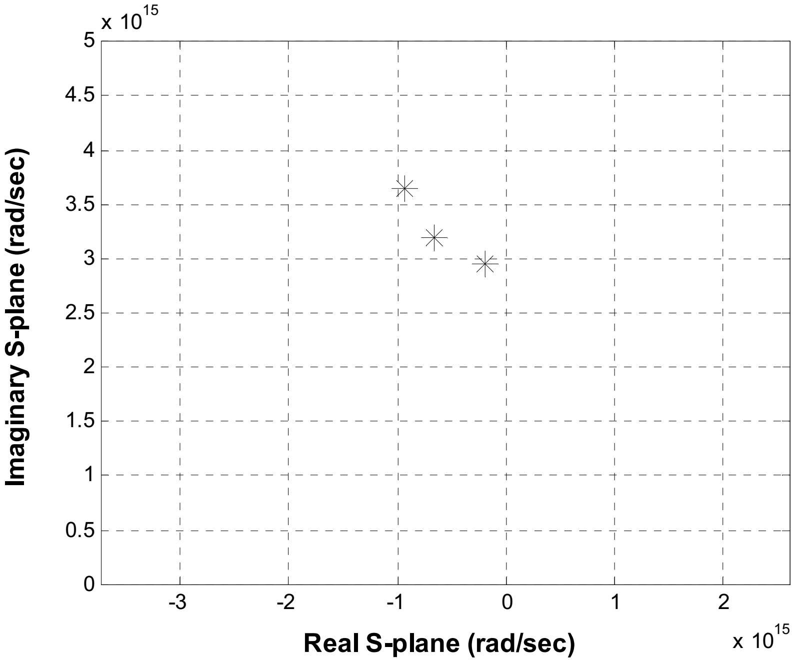

Figure 3. Pole locations in the s-plane.

Equation 7 is applicable to a general Lorentz media in which there are no zeros at the origin of the s-plane. There is a little modification required to equation 7 as we have two additional zeros at the origin as indicated by equation 2. Therefore, equation 7 becomes

$$S_k^n = 2e^{\alpha\Delta t}\cos(\beta\Delta t)\,S_k^{n-1} - e^{2\alpha\Delta t}S_k^{n-2} + \frac{u_o\omega_{pk}^2}{\beta_k}e^{\alpha\Delta t}(\sin(\varphi) + \sin(\beta\Delta t)\cos(\varphi) - \cos(\beta\Delta t)\sin(\varphi))\,H^n\Delta t \qquad (8)$$

where

$$\varphi = tan^{-1}\left(-\frac{2\alpha\beta}{\alpha^2-\beta^2}\right) \qquad (9)$$

The update equation 6 is still applicable.

4. NUMERICAL RESULTS

For simplicity both permittivity and permeability are modeled by the same multi-pole dispersive media. The phase velocity is calculated and is shown in figure 4. The figure shows an excellent agreement between the FDTD results and the analytical values. Within the desired band, the phase velocity is equal to negative of free space velocity. Thus, the metamaterial is matched to free space from about 3.2 x10^{15} rad/sec to about 4.2x10^{15} rad/sec. It is also interesting to note that the permeability remains less then one for a very large band, which indicates the possibility of designing a very broad band material lighter than free space.

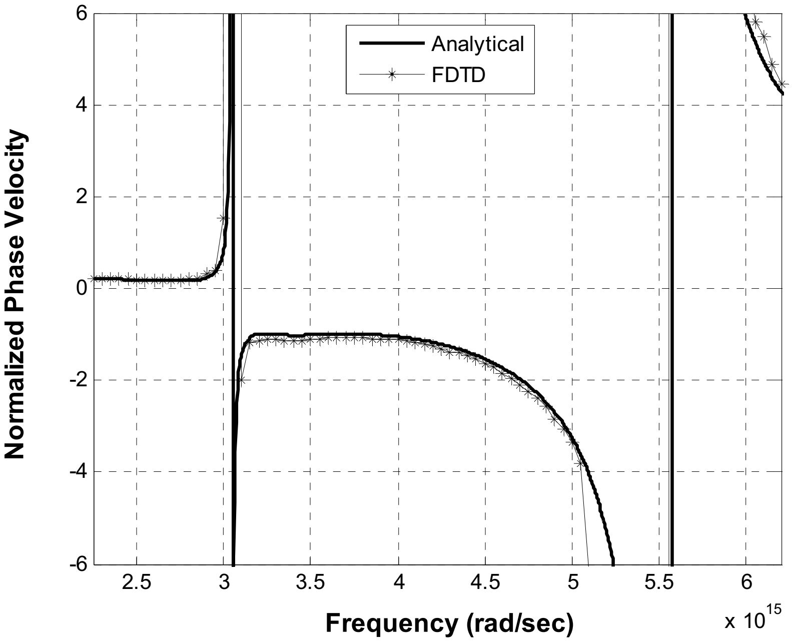

Figure 4. Phase velocity normalized to free space value.

5. CONCLUSIONS

A multi-pole dispersive model based on the split ring resonators has been proposed for the development of a broad band metamaterial at the optical frequencies. The proposed media was simulated using the FDTD z-transform method. It has been shown that such media offers much better results regarding the frequency response as compared to that of a sing SRR medium. It is worth noting, however, that the media is lossy and some means of overcoming the loss must be investigated.

ACKNOWLEDGEMENT

The authors would like to acknowledge the support provided by King Fahd University of Petroleum and Minerals.

REFERENCES

1. Pendry, J. B., A. J. Holden, W. J. Stewart and I. Young, “Extremely Low Frequency Plasmons in Metallic Macrostructures” *Physical Review Letters*, Vol. 76, No. 25, 4773-4776, 1996.
2. Pendry, J. B., A. J. Holden, D. J. Robbins and W. J. Stewart, ” Magnetism from Conductors and Enhanced Nonlinear Phenomena” *IEEE transactions on Microwave Theory and Techniques*, vol. 47, no. 11, 2075-2084, 1999.
3. Ziolkowski, R. W. and E. Heyman, “Wave propagation in media having negative permittivity and permeability” *Physical Review E*, Vol 64, 2001.

4. Qiang, R. and Ji Chen, "Modeling electrical properties of gold films at infrared frequencies using FDTD method" *International journal of Infrared and Millimeter Waves*, Vol 25, No 8, 2004.
5. Taflove, A., *Computational Electrodynamics*, Norwood, MA: Artech House, 1995.