Optical properties of metallic films for vertical-cavity optoelectronic devices

Aleksandar D. Rakic, Aleksandra B. Djurisic, Jovan M. Elazar, and Marian L. Majewski

We present models for the optical functions of 11 metals used as mirrors and contacts in optoelectronic and optical devices: noble metals (Ag, Au, Cu), aluminum, beryllium, and transition metals (Cr, Ni, Pd, Pt, Ti, W). We used two simple phenomenological models, the Lorentz–Drude (LD) and the Brendel–Bormann (BB), to interpret both the free-electron and the interband parts of the dielectric response of metals in a wide spectral range from 0.1 to 6 eV. Our results show that the BB model was needed to describe appropriately the interband absorption in noble metals, while for Al, Be, and the transition metals both models exhibit good agreement with the experimental data. A comparison with measurements on surface normal structures confirmed that the reflectance and the phase change on reflection from semiconductor–metal interfaces (including the case of metallic multilayers) can be accurately described by use of the proposed models for the optical functions of metallic films and the matrix method for multilayer calculations. © 1998 Optical Society of America

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1. Introduction

It is well known that the performances of optoelectronic devices are strongly influenced by the choice of their ohmic contacts. Recently semiconductor lasers and light-emitting diodes with metallic mirrors have received much attention.

In vertical-cavity-surface-emitting lasers (VCSEL’s) a metallic layer is commonly deposited on top of a semiconductor distributed Bragg reflector (DBR) mirror to increase the reflectivity of the mirror and to serve as a contact.1–5 Asymmetric cladding separate confinement heterostructure lasers rely on the surface metal to form an adequate waveguide for transverse confinement of the optical mode. Consequently, the modal properties are a strong function of the choice of metals (and metal thickness) for the p-type contact.6–8 Metallic layers are not only interesting for lasers, they are also useful in a new generation of optical devices called resonant-cavity light-emitting diodes (RCLED’s). The RCLED’s have structures similar to VCSEL’s, but they require mirror reflectivity of only 90–95%. This reflectivity can be achieved by using the metallic layer instead of the technologically more involved semiconductor DBR mirror. RCLED’s have been demonstrated with either one9–11 or both metallic mirrors.12,13 The RC scheme was also successfully applied to photodetectors with metallic (Ag, Al) mirrors.14,15

In all cases the needed high mirror reflectivity can be attained by using pure noble metals or Al, whereas the good ohmic contact and reliable adhesion are achieved by using layers of wetting metals (Ti, Pd, Cr) between the semiconductor and the high-reflectivity metal layer. For metal/GaAs ohmic contacts, the preferred metal is usually Au. Unfortunately, a pure Au contact has two major disadvantages; it has very poor adhesion and it diffuses rapidly into the semiconductor.6 Several other metallization schemes for GaAs devices are also in use: nonalloyed AuZn/Au for p-type ohmic contact and alloyed AuGe/Ni/Au for n-type ohmic contact,13 Ag/Au/Ti/Au for p-type and AuGe for n-type contact,10 Ag/CdSnO for p-type semitransparent contacts.11 The metal/InP ohmic contact design concepts are very similar to those suggested for metal/GaAs systems. Traditionally, Au-based alloy contacts (AuZn, AuZnNi, AuBe, AuGe, AuNi, AuCr, AuCd, AuMg, AuMn) have been used as well as nonalloyed Ti/Pt/Au and Ti/Au metallization schemes for both contacts.14,16,17 How-
ever, nonalloyed composite coatings are much preferred when contact serves also as part of the optical cavity. Materials such as W and W alloys are certainly suitable candidates for obtaining abrupt metal–semiconductor interfaces. Pure gold contacts, acting at the same time as semitransparent mirrors, have been found suitable for CdHgTe and CdZnTe devices.9

Because metallic mirrors are the elements of the resonant cavity of VCSEL’s and RCLED’s, it is necessary to determine precisely the phase and the amplitude of the reflection coefficient of such a metal–semiconductor interface to design the resonator properly. Taking into account the phase change on reflection on the metal–semiconductor interface enables one to calculate the thickness of the last semiconductor layer (the phase-matching layer) before the metal to match the phase characteristic of DBR at the design wavelength. The phase change on reflection at the metal–semiconductor interface in the near-IR and visible wavelength range is always less than 180°. Furthermore it can be a very sensitive function of wavelength in the vicinity of interband transitions of the employed metal (see, for example, Ref. 18 and references therein).

The amplitude and the phase of the reflection coefficient can be calculated from the optical constants [refractive index \( n(\omega) \) and extinction coefficient \( k(\omega) \)] or from the complex optical dielectric function \( \varepsilon_r(\omega) = \varepsilon_{r1}(\omega) - i\varepsilon_{r2}(\omega) \). Often models for \( \varepsilon_r(\omega) \) that are closely related to the electronic band structure are not amenable to such interpretation. Therefore simple phenomenological models such as the Lorentz–Drude (LD) oscillator model20 or Erman’s model,21 based on the damped harmonic oscillator approximation, which can be used to describe optical properties of an arbitrary solid, are still frequently used.22–24 Although there are several compilations of the optical properties of metals,25–30 authors know of no systematic parameterization, other than studies in which the free-electron Drude model was used, which is not adequate in the near-IR and the visible wavelength range. Therefore there is need for a study that would provide a description of the optical functions of metals in terms of a simple model and that would cover all the metals used for optoelectronic applications.

We present parameterization of the large number of metals for optoelectronic applications, using (a) a simple LD model and (b) a recent model by Brendel and Bormann31 (BB). We also give a new derivation of this flexible phenomenological model and present a BB model in two different analytic forms suitable for implementation on a computer. Therefore the purpose of this paper is twofold: (a) to present a comprehensive and accurate parameterization of the optical properties of metals for optoelectronic applications in a wide spectral range and (b) to compare the LD model with the new BB model on several examples.

In Section 2 we describe the LD and the BB models. In Section 3 we give an overview of the optical dielectric functions of the metals used in this study and provide parameterizations of these optical functions in terms of LD and BB models. In Section 4 we compare our results with recent measurements performed on vertical cavity structures. In Appendix A we outline the derivation of our analytic form of the BB model.

2. Material Models

A. Lorentz–Drude Model

First we briefly discuss the LD model often used for parameterization of the optical constants of metals.18,32–34 It has been shown35,36 that a complex dielectric function \( \varepsilon_r(\omega) \) can be expressed in the following form:

\[
\varepsilon_r(\omega) = \hat{\varepsilon}_r^{(i)}(\omega) + \hat{\varepsilon}_r^{(b)}(\omega),
\]

which separates explicitly the intraband effects (usually referred to as free-electron effects) from interband effects (usually referred to as bound-electron effects). The intraband part \( \hat{\varepsilon}_r^{(i)}(\omega) \) of the dielectric function is described by the well-known free-electron or Drude model37,38:

\[
\hat{\varepsilon}_r^{(i)}(\omega) = 1 - \frac{\Omega_p^2}{\omega(\omega - i\Gamma_0)}.
\]

The interband part of the dielectric function \( \hat{\varepsilon}_r^{(b)}(\omega) \) is described by the simple semiquantum model resembling the Lorentz result for insulators:

\[
\hat{\varepsilon}_r^{(b)}(\omega) = \sum_{j} \frac{\Gamma_j f_j \omega_p^2}{(\omega^2 - \omega_j^2 + i\omega\Gamma_j)},
\]

where \( \omega_p \) is the plasma frequency, \( k \) is the number of oscillators with frequency \( \omega_j \), strength \( f_j \), and lifetime \( 1/\Gamma_j \), while \( \Omega_p = \sqrt{\hbar \omega_p} \) is the plasma frequency associated with interband transitions with oscillator strength \( f_0 \) and damping constant \( \Gamma_0 \).

B. Brendel–Bormann Model

Let us now concentrate on the interband part \( \hat{\varepsilon}_r^{(b)}(\omega) \) of the dielectric function. Usually the Lorentz model employs oscillators at major critical points (CP’s) in the joint density of states that correspond to interband transition energies \( \hbar \omega_{ij} \), with some additional oscillators to model absorption between CP’s. It has been shown39,40 however, that usually the Gaussian line shape is a much better approximation for the broadening function than the Lorentzian line shape. If the same strength and full width at half-maximum are assumed for both absorption line shapes, the Lorentzian broadening function has higher and more extended wings compared with the Gaussian one. Accordingly, all the models based on the Lorentzian broadening function exhibit excessive absorption far from the CP’s. Recently, Brendel and
Bormann\textsuperscript{31} have proposed a model for a dielectric function of solids that replaces a Lorentz oscillator with a superposition of an infinite number of oscillators, given by

\[ \chi_j(\omega) = \frac{1}{2\pi \sigma_j} \int_{-\infty}^{\infty} \exp\left[ -\frac{(x - \omega)^2}{2\sigma_j^2} \right] \frac{f_j \omega}{(x^2 - \omega^2) + i\omega \sigma_j} \, dx. \]  

The number of harmonic oscillators per frequency interval is determined by a Gaussian function. In their original paper\textsuperscript{31} the authors suggested that the model is applicable to an amorphous solid in the far-IR part of the spectrum and constructed the model from a phenomenological point of view. However, we show that this model can be used to describe optical properties of a wide range of materials including metals. Integral (4) can be solved analytically (see Appendix A) to obtain

\[ \chi_j = \frac{i\sqrt{\pi} f_j \omega_{j0}^2}{2 \sqrt{2a_j \sigma_j}} \left[ \frac{w(a_j - \omega)}{\sqrt{2\sigma_j}} + \frac{w(a_j + \omega)}{\sqrt{2\sigma_j}} \right], \]  

where \( a_j = a_{j'} + ia''_j = (\omega^2 - i\omega \Gamma_j)^{1/2} \) and \( a_{j'} > 0 \), while \( w(z) \) is the error integral of the complex argument, defined by [see Ref. 41, Eq. (7.1.3)]

\[ w(z) = e^{-z^2} \text{erf}(-iz) \quad \text{Im}[z] > 0, \]  

and \( \text{erf}(z) \) is the complementary error function,

\[ \text{erf}(z) = \frac{2}{\sqrt{\pi}} \int_{-z}^{\infty} \exp(-t^2) \, dt. \]  

A solution satisfying the condition \( a_{j''} > 0 \) is

\[ a_{j'} = \frac{\omega}{\sqrt{2}} \left[ \left(1 + (\Gamma_j/\omega)^2\right)^{1/2} + 1 \right]^{1/2}, \]

\[ a_{j''} = \frac{\omega}{\sqrt{2}} \left[ \left(1 + (\Gamma_j/\omega)^2\right)^{1/2} - 1 \right]^{1/2}. \]  

We suggest that Eq. (5) also be expressed in terms of confluent hypergeometric functions, namely, the Kummer functions of the second kind \( U(a, b, z) \). By employing the relation between \( \text{erf}(z) \) and \( U(a, b, z) \) [cf. Ref. 41, Eq. (13.6.39)],

\[ U(1/2, 1/2, z^2) = \sqrt{\pi} e^{z^2} \text{erf}(z), \]  

we obtain

\[ \chi_j = \frac{if_j \omega_{j0}^2}{2 \sqrt{2a_j \sigma_j}} \left\{ U\left[ 1/2, 1/2, -\frac{(a_j - \omega)}{2\sigma_j} \right] + U\left[ 1/2, 1/2, -\frac{(a_j + \omega)}{2\sigma_j} \right] \right\}. \]  

When we keep in mind that special functions of complex arguments are now readily available through languages such as Mathematica,\textsuperscript{42} or from various Fortran and C libraries, this model is as easy to implement as a simple LD model. The expression for the complete optical dielectric function now reads

\[ \hat{\epsilon}_j(\omega) = 1 - \frac{\Omega_0^2}{\omega(\omega - i\gamma_j)} + \sum_{j=1}^{k} \chi_j(\omega), \]  

where \( k \) is the number of BB oscillators used to interpret the interband part of the spectrum. Thus an analytic function is obtained that satisfies the Kramers–Kronig (K–K) relations and has a flexible shape of the absorption profile. This function, however, introduces an additional parameter per each transition besides the strength, energy, and linewidth. This additional parameter \( \sigma \) allows for a continuous change in the absorption line shapes ranging from a purely Lorentzian for \( \sigma_j \approx 0 \) to a nearly Gaussian for \( \Gamma_j \approx 0 \). Therefore a range of absorption function approximations with similar kernels and different wings can be obtained with Eq. (11).

C. Optical Constants and Reflectivity

Refractive index \( n \) and extinction coefficient \( k \), being the real and the imaginary parts of the complex refractive index \( N = n - ik \), can be calculated from the complex optical dielectric function:

\[ n = \frac{1}{\sqrt{2}} \left[ (\epsilon_{21}^2 + \epsilon_{22}^2)^{1/2} + \epsilon_{21} \right]^{1/2}, \]

\[ k = \frac{1}{\sqrt{2}} \left[ (\epsilon_{21}^2 + \epsilon_{22}^2)^{1/2} - \epsilon_{21} \right]^{1/2}. \]  

For the simple semiconductor–metal interface, normal incidence reflectivity can be calculated by use of\textsuperscript{43}

\[ R = \frac{(n_0 - n_1)^2 + k_1^2}{(n_0 + n_1)^2 + k_1^2}, \]  

where the phase shift associated with the reflected beam is\textsuperscript{43}

\[ \varphi = \arctan \frac{2n_0k_1}{(n_0^2 - n_1^2 - k_1^2)}. \]  

and where \( n_0 \) is the refractive index of the incident medium (semiconductor) and \( N_1 = n_1 - i k_1 \) is the refractive index of the substrate (metal). Equations (13) and (14) imply that the incident medium is lossless owing to the well-known problems of defining reflectivity for the wave incident from the lossy medium.\textsuperscript{43} However, the amplitude reflection coefficient and the phase are well defined, and there are no problems in calculating the phase shift upon reflection needed in resonator design, even if the lossy incident medium (semiconductor) is assumed.\textsuperscript{43,44}

3. Results for 11 Metals

We have parameterized 11 most frequently used metals: noble metals (Ag, Au, and Cu), aluminum, beryllium, and some of the transition metals (Cr, Ni, Pd, Pt, Ti, and W). For all the metals we have carefully chosen relevant optical data, trying always to engage
the measurements obtained on films similar (in terms of geometry, morphology, and the process of growth) to the films used in optoelectronic devices. Also, wherever possible, we used K–K consistent sets of optical data in the whole range. To fit the models (LD and BB) to the data, we employed the following objective function:

$$\chi^2 = \sum_{i=1}^{i=N} \left[ \frac{\varepsilon_{1r}(\omega_i) - \varepsilon_{1r}^{\exp}(\omega_i)}{\varepsilon_{1r}^{\exp}(\omega_i)} \right] + \left[ \frac{\varepsilon_{2r}(\omega_i) - \varepsilon_{2r}^{\exp}(\omega_i)}{\varepsilon_{2r}^{\exp}(\omega_i)} \right]^2$$

To minimize the merit (or cost) function, we use the acceptance-probability-controlled simulated annealing algorithm (described in detail in Refs. 32 and 34), which was specifically designed for solving parameter estimation problems. The algorithm has two independent features that make it superior to other simulated annealing algorithms in terms of convergence rate and robustness: first, the replacement of the temperature control of the cooling schedule with direct control of acceptance probability; second, the highly efficient move generation mechanism. This algorithm has proved to be insensitive to initial parameter values and extremely efficient in escaping local minima. No attempt has been made here to constrain the values of the oscillator strengths and linewidths in the fitting procedure, even by an order of magnitude. Optimal values for LD model parameters, obtained by this algorithm, are given in Table 2, whereas Table 3 presents optimal parameters for the BB model. The values for oscillator strengths correspond to the plasma frequencies $\hbar\omega_p$ given in Table 1.

A. Silver

For parameterization of the optical constants of silver, we used the data tabulated in Ref. 45. This composite set consists of the polarimetric data of Dold and Mecke46 between 0.125 and 0.98 eV, polarimetric measurements of Winsemius et al.47 between 0.65 and 3.3 eV, and K–K analyzed in situ reflectance measurements of Leveque et al.48 from 3.3 to 26.5 eV. For our fits we used data between 0.125 and 6 eV.

The data of Leveque et al.46 and of Winsemius et al.47 agree well at ~3.4 eV where they meet. The dispersion data ($\varepsilon_{r1}$) of Dold and Mecke46 have the same slope but slightly lower values than the data of Winsemius et al.47 in the region of overlap (0.65–1 eV). The absorption data ($\varepsilon_{r2}$) of the same groups continue well at ~1 eV with a somewhat steeper slope in the data of Winsemius et al.47. Our model dielectric function agrees well in slope and magnitude with both data sets, as depicted in Fig. 1.

B. Gold

Gold has been the subject of extensive optical studies. Of particular interest to our application is the study of Théye49 providing the dielectric function of Au in the 0.5–6-eV range from the measurements of the reflectance and the transmittance of semitransparent

<table>
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<td>Be</td>
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<td>Cr</td>
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<td>Ni</td>
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<tr>
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<td>Ti</td>
<td>7.29</td>
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<tr>
<td>W</td>
<td>13.22</td>
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Table 1. Values of the Plasma Frequencies $\hbar\omega_p$ (eV)

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<th>$f_0$</th>
<th>$\Gamma_0$</th>
<th>$f_1$</th>
<th>$\Gamma_1$</th>
<th>$\omega_1$</th>
<th>$f_2$</th>
<th>$\Gamma_2$</th>
<th>$\omega_2$</th>
<th>$f_3$</th>
<th>$\Gamma_3$</th>
<th>$\omega_3$</th>
<th>$f_4$</th>
<th>$\Gamma_4$</th>
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<th>$\omega_5$</th>
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<td>0.065</td>
<td>0.816</td>
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<td>9.083</td>
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<td>2.419</td>
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<tr>
<td>Au</td>
<td>0.760</td>
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<td>4.384</td>
<td>2.214</td>
<td>13.32</td>
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<tr>
<td>Cu</td>
<td>0.575</td>
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<td>0.227</td>
<td>0.291</td>
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<td>1.056</td>
<td>2.957</td>
<td>11.18</td>
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<td>4.384</td>
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<tr>
<td>Al</td>
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<td>0.031</td>
<td>0.162</td>
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<td>2.957</td>
<td>11.18</td>
<td>4.304</td>
<td>4.384</td>
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<td>Cr</td>
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<td>0.100</td>
<td>0.121</td>
<td>0.150</td>
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<td>1.544</td>
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<td>Ni</td>
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Table 2. Values of the LD Model Parameters

*In electron volts.*
films of various thicknesses. A detailed analysis of these samples indicated that it is possible to produce very thin but continuous films with the same properties as the bulk material. The results on well-crystallized films of thicknesses from 10 to 25 nm evaporated in ultrahigh vacuum \( \sim 10^{-11} \) Torr show good agreement with data obtained on bulk samples.49

In our analysis we used the ellipsometric data of Dold and Mecke46 in the 0.125–0.98-eV region and the data of Théye49 between 1.0 and 6.0 eV. Both data sets were tabulated in Ref. 45. Figure 2 shows the real and the imaginary part of the dielectric function of gold versus photon energy between 0.2 and 5 eV. At \( \sim 2 \) eV, where the lowest-laying interband transitions occur in Au, the BB model fits the data much better than the LD model. This better set can

### Table 3. Values of the BB Model Parameters

<table>
<thead>
<tr>
<th>Metal</th>
<th>Parameter</th>
<th>Ag</th>
<th>Au</th>
<th>Cu</th>
<th>Al</th>
<th>Be</th>
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<tr>
<td>( \Gamma_0 )</td>
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<td>0.050</td>
<td>0.030</td>
<td>0.037</td>
<td>0.035</td>
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<tr>
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<td>0.066</td>
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<td>( \sigma_1' )</td>
<td>2.025</td>
<td>0.218</td>
<td>0.416</td>
<td>0.163</td>
<td>0.131</td>
<td>0.281</td>
<td>0.317</td>
<td>0.066</td>
<td>0.782</td>
<td>1.459</td>
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<tr>
<td>( f_2 )</td>
<td>0.133</td>
<td>0.050</td>
<td>0.081</td>
<td>0.060</td>
<td>0.067</td>
<td>0.261</td>
<td>0.039</td>
<td>0.093</td>
<td>0.665</td>
<td>0.218</td>
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<tr>
<td>( \Gamma_2 )</td>
<td>0.067</td>
<td>0.035</td>
<td>0.047</td>
<td>0.315</td>
<td>3.962</td>
<td>3.957</td>
<td>0.120</td>
<td>0.497</td>
<td>1.851</td>
<td>0.100</td>
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<td>( \omega_2 )</td>
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<td>2.849</td>
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<td>0.469</td>
<td>0.584</td>
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<td>1.317</td>
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<td>0.349</td>
<td>0.469</td>
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<td>( f_3 )</td>
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<td>0.312</td>
<td>0.324</td>
<td>0.182</td>
<td>0.346</td>
<td>0.817</td>
<td>0.127</td>
<td>0.309</td>
<td>0.551</td>
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<td>0.113</td>
<td>1.587</td>
<td>2.398</td>
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<td>0.799</td>
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<tr>
<td>( \Gamma_5 )</td>
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<tr>
<td>( \omega_5 )</td>
<td>18.56</td>
<td>27.97</td>
<td>—</td>
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<tr>
<td>( \sigma_5 )</td>
<td>0.516</td>
<td>1.795</td>
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*aIn electron volts.*

In electron volts.

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Fig. 1. Real and imaginary parts of the optical dielectric function of Ag: solid curves, values that we calculated using the BB model; dashed curves, the LD model. Also shown are the selected experimental data points from Dold and Mecke,46 Winsemius et al.,47 and Leveque et al.48

Fig. 2. Real and imaginary parts of the optical dielectric function of Au: solid curves, values that we calculated using the BB model; dashed curves, the LD model. Also shown are selected experimental data points from Dold and Mecke46 and Théye.49

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be attributed to the Gaussian broadening in the interband part of the spectrum, which becomes dominant in this region.

C. Copper

Data for the copper that we used in this research are from the studies of Hagemann et al.\textsuperscript{50} and Ordal et al.\textsuperscript{26} The composite data set reported in Ref. 50 was obtained by combining original transmission measurements, performed on films evaporated in the conventional vacuum, with data from various sources\textsuperscript{36,51–53} to provide an absorption spectrum large enough for a K–K analysis. Results of this analysis are tabulated in Refs. 45 and 25. The data of Ordal et al.\textsuperscript{26} pertinent in the IR and far-IR range, are obtained from the K–K analysis of their reflectance data. We used the data of Ordal et al.\textsuperscript{26} from 0.1 to 0.87 eV and data of Hagemann et al.\textsuperscript{50} from 1 to 6 eV. Figure 3 depicts the real and the imaginary part of the dielectric function as a function of energy (open circles, tabulated data; solid curves, the BB model; dashed curves, the LD model).

An interesting new result is the surface plasmon–polariton study of Nash and Sambles.\textsuperscript{54} They thermally deposited copper film, 350 nm thick, onto the high-quality silica grating with a well-defined pitch (800 nm) and a profile. Film was covered by a 50-nm-thick layer of silicon monoxide without being removed from vacuum. In Fig. 3 we compare several experimental data sets for Cu, showing that the values of Nash and Sambles for $\varepsilon_{1}$ closely agree with the data of Hagemann et al.\textsuperscript{50} and Ordal et al.\textsuperscript{26} in the region of overlap. The values of Nash and Sambles for $\varepsilon_{2}(0)$ are smaller than the majority of other data in the visible range; authors have claimed that this is due to the absence of contamination in their protected copper films.\textsuperscript{54}

D. Aluminum

To fit the models to data for Al we used results from a recent study of Rakic.\textsuperscript{18} In Ref. 18 optical and electron-energy-loss data for evaporated aluminum films have been critically analyzed and used in an iterative, self-consistent algorithm presenting the combination of the K–K analysis and the LD model application. New values for the optical functions of aluminum have been reported in the wide spectral range from 200 $\mu$m (6.2 meV) to 0.12 nm (10 keV). These functions are in accordance with recent calculations of Lee and Chang,\textsuperscript{55} dc conductivity measurements, and are in good agreement with both the peak position and the linewidth of the electron-energy-loss data. The results are examined for internal consistency by inertial and $f$-sum rules. Figure 4 shows excellent agreement between tabulated data\textsuperscript{18} (open circles) and models (solid curves, BB model; dashed curves, LD model). Recent research of Nguyen et al.\textsuperscript{56} shows that for high-rate-evaporated aluminum the electron mean free path increases by nearly an order of magnitude over the thickness range of 5.5–6 nm, representing a transition of the particle film in the continuous film with single-crystalline grains that span the thickness of the film. For thicknesses over the 6-nm parallel-band, transitions that dominate the bulk-material interband spectrum are noticeable. Keeping in mind that aluminum layers used in optoelectronic devices are usually more than 10 nm thick, we expect the bulk dielectric function of aluminum presented here to be a reasonably good approximation for aluminum thin-film dielectric function.

E. Beryllium

There have been only a few studies of optical properties of Be, and they produced different results, espe-
cially in the visible range of the spectrum. The inconsistency in the results can be attributed to the difficulty of sample preparation. The data that we used for fitting are from the excellent study of Arakawa et al.\textsuperscript{57} They obtained a consistent set of optical constants by performing a K–K analysis in the wide spectral range from 0.06 to 26 eV, using normal-incidence reflectance data from various sources.

Figure 5 shows the real and the imaginary part of the optical dielectric function as a function of energy. Open circles represent data from Ref. 57; the solid curves, the BB model; and the dashed curves, the LD model.

F. Chromium

In our research we used the K–K-analysis data of Bos and Lynch\textsuperscript{58} tabulated in Ref. 59, and the bulk-sample ellipsometric data of Kirillova and Noskov\textsuperscript{60} tabulated in Ref. 29. The measurements of Bos and Lynch\textsuperscript{58} extend to 0.095 eV in the IR, although their K–K analysis extends further. We used their data between 0.46 and 5 eV. Kirillova and Noskov\textsuperscript{60} reported measurements in the 0.069–4.9-eV range. We used their data between 0.069 and 0.41 eV.

The frequently quoted data of Johnson and Christy\textsuperscript{61} measured in the 0.64–6.60-eV range, show lower reflectance values, probably due to the fine-grain structure and voids in the films. One result of their research, which is particularly important to our application, addresses the change in optical constants with film thickness. The optical constants of Cr measured on two films, with thicknesses of 40 and 50 nm, agreed with each other well within the estimated experimental error.\textsuperscript{61} However, measurements on 36- and 26-nm-thick films showed a slight increase in \(n\) with decreasing film thickness, whereas the value of \(k\) was almost unaffected. Figure 6 shows our calculated values for the real and the imaginary part of the dielectric function of chromium. Good agreement with experimental data can be observed for both models.

G. Nickel

Nickel does not behave as a free-electron metal for wavelengths shorter than \(\sim 20 \mu\text{m}\); probably because of this agreement among the published optical functions of Ni is good. Here we used data from the research of Lynch and Hunter.\textsuperscript{45} This consistent set of optical functions is obtained by K–K inversion of reflectance and absorptance data from a number of sources in the range from 0.08 to 50 eV. For our fits we have employed data from 0.2 to 5 eV. Figure 7 shows the real and the imaginary part of the optical dielectric function versus photon energy (open circles, tabulated data; solid curves, BB model; dashed curves, LD model). Good agreement between tabulated data and both models can be observed.

H. Palladium

The optical properties of palladium have been the subject of several investigations. Unfortunately, results disagree as to the magnitude of the optical functions, depending strongly on sample preparation and the method for the analysis of experimental data.

In this paper we used the data from Weaver and Benbow\textsuperscript{62} in the 0.1–0.6-eV range and from Johnson and Christy\textsuperscript{61} between 0.6 and 6 eV. These data are also tabulated in the review of Borghesi and Piaggi.\textsuperscript{63} Johnson and Christy\textsuperscript{61} reported the systematic dependence of optical constants on film thickness. Analyzing films 16.7, 25.2, 32.4, and 41.2 nm thick, they noted that the values of the conductivities increased with film thickness. In the case of the three thickest films, the successive differences were not more than the assumed experimental error. This indicates
that, for thicknesses of more than 20 nm, bulklike values of optical constants represent a good approximation. Figure 8 shows excellent agreement between our fits (for both models) and tabulated values of the real and the imaginary part of the optical dielectric function.

I. Platinum

To fit our model to data, we used the tabulation of the optical constants of Pt given in 45 based on the study of Weaver.64 Weaver used reflectance65–67 and transmittance68 data from a number of sources to obtain \( n \) and \( k \) by the K–K technique. Interband transitions for platinum are expected, according to Ref. 69, at \( \sim 6.3, 7.8, 9.3, \) and 10.8 eV. The structure of optical constants64 is evident at approximately 7.4 and 9.8 eV. As the other transition metals do, platinum possesses the characteristic minimum in \( \epsilon_{Ext} \) with an additional structure at higher energy. Figure 9 shows a dielectric function versus energy for platinum (open circles, tabulated data; solid curves, BB model; dashed curves, LD model).

J. Titanium

Johnson and Christy61 investigated the reflectivity and the transmissivity of thin titanium films (29.8–35 nm thick) evaporated in a conventional vacuum from a tungsten filament. A fast evaporation rate was used to insure the surface smoothness of the films. Lynch et al.70 reported reflectivity and absorptivity measurements of mechanically polished and electropolished bulk Ti samples in the energy range from 0.1 to 30 eV. K–K analysis was used to invert the measured spectrum to the optical dielectric function. This data set is tabulated in Ref. 71 (pp. 12–145), but owing to an error, instead of Ref. 70 the paper by Johnson and Christy61 is quoted as a source of data. Kirillova and coworkers72–74 used an ellipsometric technique to measure the optical constants of mechanically and chemically polished polycrystalline samples from 0.06 to 2.6 eV.

The \( \epsilon_{Ext} \) values from Kirillova and coworkers are in reasonably good agreement with the data of Johnson and Christy61 in the region of overlap, although all the energies of the structures do not agree well. The \( \epsilon_{Ext} \) values from Kirillova and coworkers are noticeably lower (for photon energies greater than 1 eV). The prominent maximum in the \( \epsilon_{Ext} \) values of Lynch et al.70 at \( \sim 0.46 \) eV is absent in the data of Kirillova and coworkers.72–74 The weaker peak in the \( \epsilon_{Ext} \) values of
Lynch et al. located at 1.3 eV can be noticed in the data of Johnson and Christy at 1.15 eV and in the data of Kirillova and coworkers at 1 eV (see Fig. 10).

In our fits we used data from Kirillova and Charikov in the energy range from 0.06 to 2.6 eV. In Fig. 11 we compare the real and the imaginary part of the model dielectric function of Ti against the pertinent experimental data.

K. Tungsten

Most measurements of the optical functions of tungsten are in good agreement. In this paper we used the optical constants of Weaver et al. tabulated in Ref. 45. Optical constants were obtained by the K–K inversion of reflectivity data in combination with the absorption spectrum from Haensel et al. Figure 12 shows the real and the imaginary part of the dielectric function of W versus photon energy (open circles, tabulated data; solid curves, BB model; dashed curves, LD model).

4. Comparisons and Conclusions

If one treats a metal as a perfect conductor, one needs only to enforce the tangential component of the electric field \( E_{\text{tan}} = 0 \) on its surface since there is no field penetration into it. Material response is \( \pi \) out of phase with respect to the electric field incident on the metal. This phase difference of a half-period is a delay between the time when the incident field is absorbed and the time when it is reradiated by the medium. Interference between these two fields causes the high reflectivity of the metal. In the IR and visible wavelength range, the phase change on reflection from real metal is always less than \( \pi \) and can be strongly frequency dependent in the vicinity of the interband transition frequencies. Consequently, the reflectivity \( R \) is also less than one. To illustrate this we calculated \( R(\omega) \) and \( \varphi(\omega) \) for four semiconductor–metal interfaces in the spectral range from 0.1 to 6 eV (12.4–0.2 \( \mu \)m). We employed models for the optical constants of metals \( [N(\omega) = n(\omega) - ik(\omega)] \) described in Section 3, and for GaAs we used our model for \( N(\omega) = n(\omega) - ik(\omega) \) described in Ref. 76. Figure 13 shows reflectivities for GaAs–Ag, GaAs–Au, GaAs–Al, and GaAs–Pt interfaces. Figure 14 depicts the phase change on reflection for the same four interfaces in the same wavelength range.

Let us now compare the DBR mirror terminating on air [Fig. 15(a)] and the one terminating on a metallic layer [Figs. 15(b) and 15(c)]. If the semiconductor layers comprising the DBR are of a quarter-
wave optical thickness (QWOT), the wave reflected from interface 1, which undergoes the phase change on reflection $\phi = \pi$ [see Fig. 15(a)], is in phase with the wave reflected from interface 2, which experiences a phase shift $\delta = -\pi$ while making a round trip through the last high-index layer (GaAs) and no phase change on reflection. The additional high-index QWOT layer (the phase-matching layer) is needed between the last semiconductor layer and the perfectly conductive metallic layer [see Fig. 15(b)] to keep the waves reflected from interfaces 1 and 2 in phase. Therefore the round-trip phase shift $-\pi$ in the phase-matching layer cancels the phase change on reflection at the semiconductor–metal interface, $\phi = \pi$. If we now consider the real metal with the complex refractive index $N = n - ik$, the phase change on reflection $\phi$ is determined by Eq. (14), where $\phi < \pi$ at all wavelengths. To keep the waves reflected from interfaces 1 and 2 in phase, one needs to have a matching layer with phase thickness $\delta$ satisfying $-2\delta + \phi = 0$ [see Fig. 15(c)]. Therefore the required phase thickness of the phase-matching layer is

$$\delta = \frac{\phi}{2} = \frac{2\pi}{\lambda_0} nd,$$

where $\lambda_0$ is the vacuum wavelength, while $n$ and $d$ are the refractive index and the physical thickness of the phase-matching layer. The needed thickness $d$ is therefore

$$d = \frac{\phi}{4n} \left( \frac{\lambda_0}{4\pi} \right) = \frac{\phi}{\pi} \text{(QWOT)}.$$

It is important therefore that we know precisely the phase change on reflection from the metal–semiconductor interface ($\phi$) to be able to calculate the thickness of the phase-matching layer. A recent paper of Babić et al.77 describes a method for direct measurement of $\phi$ that uses the specifically designed cavity. They recorded the reflectance spectra of Fabry–Perot (F–P) resonators formed by semiconductor layers clad by (a) air on one side and a dielectric on the other and (b) air on one side and a metal multilayer on the other. Reference 77 presents the measured reflectance for two F–P cavities: (a) air–GaAs (1000 nm)–wax and (b) air–GaAs (1000 nm)–Ti/Au (2 nm/170 nm)–wax. We used a transfer-matrix method to calculate the reflectivity of structures (a) and (b). Results are depicted in Fig. 16. Again, for the optical constants of metals we used the models from Section 3 and for the GaAs model described in Ref. 76. The solid curves are from the transfer-matrix calculations; the dashed curves show the experimental data from 77. The phase change on reflection at 1550 nm estimated from the experiment ($\phi_{\exp} \approx 137^\circ$) is in accordance with our calculated phase $\phi_{\text{calc}} = 139^\circ$.

In this paper we present, for the first time to our
knowledge, a comprehensive parameterization of the optical functions for 11 metals for applications in optoelectronic and optical devices: noble metals (Ag, Au, and Cu), aluminum, beryllium, and some of the transition metals (Cr, Ni, Pd, Pt, Ti, and W). We used two simple phenomenological models, namely, the LD and the BB models to interpret both the freeelectron and the interband parts of the dielectric response of real metals in the wide spectral range from 0.1 to 6 eV. Our results show that the BB model was needed for an appropriate description of the onset of interband absorption in noble metals (Ag, Au, Cu), while for aluminum and the transition metals both models exhibit good agreement with the experimental data. This is in accordance with our initial assumptions. The LD model due to the extended absorption of the Lorentz oscillators can hardly describe a sharp absorption edge, unless a large number of oscillators were used to describe that transition. In such a case it is physically justified and computationally simpler to employ a single Gaussian-like BB oscillator instead.

Although it is known that the optical constants of thick films (bulk) can differ from the optical constants of thin metallic films, the optical constants of evaporated films still provide a good approximation. For all the metals we carefully chose relevant optical data, trying always to use the measurements obtained on films similar (in terms of geometry, morphology, and the growth process) to the films used in optoelectronic devices. Comparison with the measurements on surface normal structures confirmed that the reflectance and the phase change on reflection from semiconductor–metal interfaces (including the case of metallic multilayers) can be accurately calculated by use of the proposed models for the optical functions of metals and the transfer matrix theory.

Appendix A

The Brendel–Borman model requires the integration of Eq. (4) in closed form. The integral to be solved is

\[
\chi_i(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \exp\left[-\frac{(x - \omega_j)^2}{2\sigma_j^2}\right] \times \frac{f_j\omega_p^2}{(x^2 - \omega_j^2) + i\omega\Gamma_j} \, dx. \tag{A1}
\]

We define \(a_j^2 = \omega_j^2 - i\Gamma_j\omega\) with \(\text{Im}[a_j] > 0\) and obtain

\[
\chi_i(\omega) = -\frac{f_j\omega_p^2}{\sqrt{2\pi}a_j\sigma_j} \left\{ \int_{-\infty}^{+\infty} \exp\left[-\frac{(x - \omega_j)^2}{2\sigma_j^2}\right] \frac{1}{a_j - x} \, dx \right. \\
+ \left. \int_{-\infty}^{+\infty} \exp\left[-\frac{(x - \omega_j)^2}{2\sigma_j^2}\right] \frac{1}{a_j + x} \, dx \right\} \tag{A2}
\]

Now the first integral can be written as

\[
I_1 = \int_{-\infty}^{+\infty} \exp\left[-\frac{(x - \omega_j)^2}{2\sigma_j^2}\right] \frac{1}{(a_j - \omega_j) - (x - \omega_j)} \, dx. \tag{A4}
\]

By introducing \(t = (x - \omega_j)/(\sqrt{2}\sigma_j)\) and \(z_1 = (a_j - \omega_j)//(\sqrt{2}\sigma_j)\), we find that [see Ref. 41, Eqs. (7.1.3) and (7.1.4)]

\[
I_1 = \int_{-\infty}^{+\infty} \frac{e^{-t^2}}{z_1 + t} \, dt = \frac{\pi}{i} e^{-z_1^2} \text{erfc}(-iz_1) = \frac{\pi}{i} w\left(\frac{a_j - \omega_j}{\sqrt{2}\sigma_j}\right). \tag{A5}
\]

Using a similar approach, we can solve the second integral

\[
I_2 = \int_{-\infty}^{+\infty} \exp\left[-\frac{(x - \omega_j)^2}{2\sigma_j^2}\right] \frac{1}{(a_j - \omega_j) + (x - \omega_j)} \, dx \tag{A6}
\]

by introducing \(t = (x - \omega_j)/(\sqrt{2}\sigma_j)\) and \(z_2 = (a_j + \omega_j)//(\sqrt{2}\sigma_j)\):

\[
I_2 = \int_{-\infty}^{+\infty} \frac{e^{-t^2}}{z_2 + t} \, dt = \int_{-\infty}^{+\infty} \frac{e^{-t^2}}{z_2 - t} \, dt = \frac{\pi}{i} e^{-z_2^2} \text{erfc}(-iz_2) = \frac{\pi}{i} w\left(\frac{a_j + \omega_j}{\sqrt{2}\sigma_j}\right). \tag{A7}
\]
If Eqs. (A5) and (A7) are finally substituted into Eq. (A1), we obtain the result

\[
\chi^\nu_i = i \pi \frac{\rho_j \omega_n^2}{2 \alpha_i \alpha_j} \left[ \frac{w(i - \omega)}{\sqrt{\alpha_i}} + \frac{w(i + \omega)}{\sqrt{\alpha_j}} \right]. \tag{A8}
\]

References


43. H. A. Macleod, Thin-Film Optical Filters (Adam Hilger, Bristol, UK, 1986).


