Electromagnetics

EE 340

Lecture 1 - Introduction

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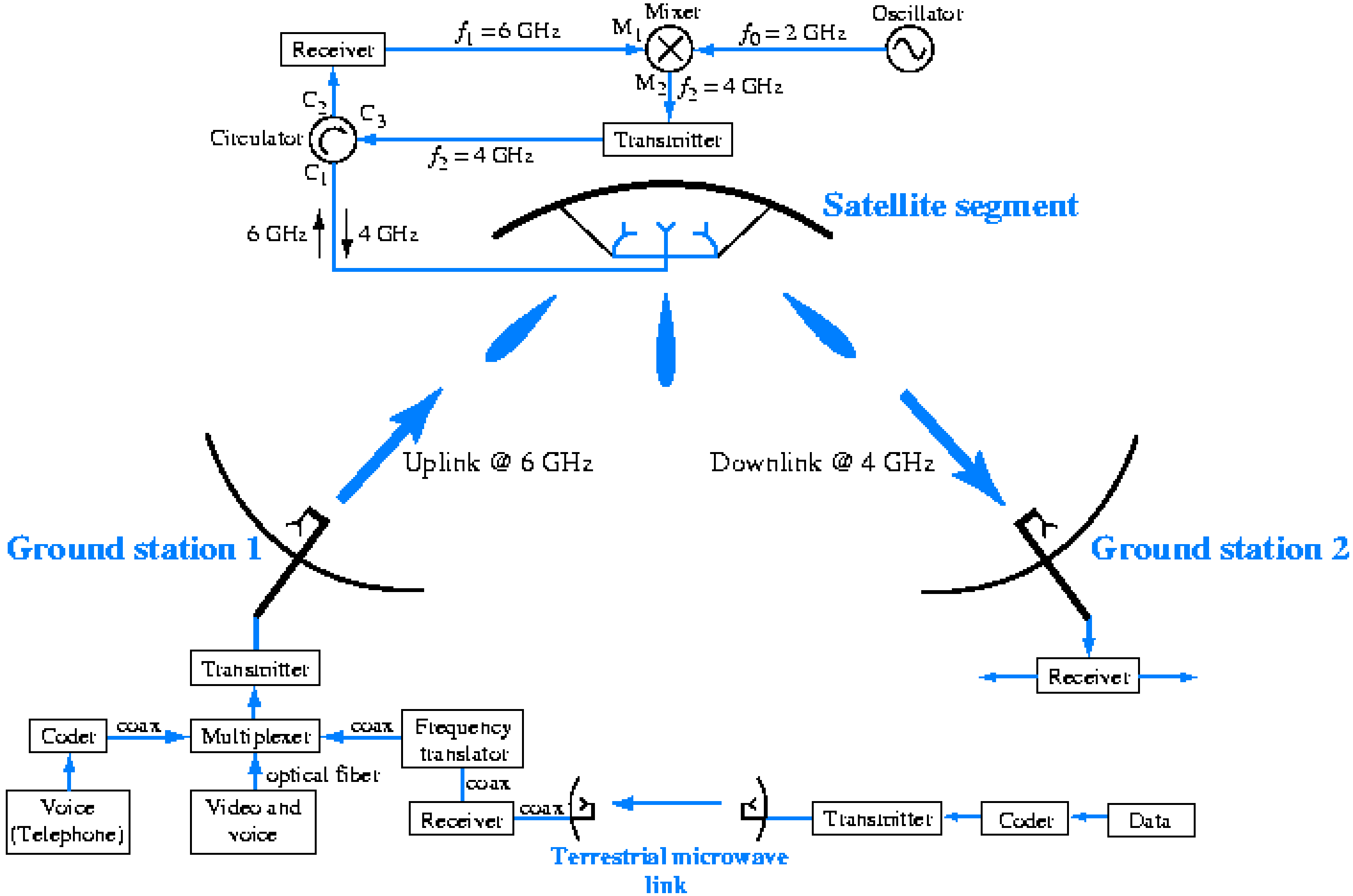
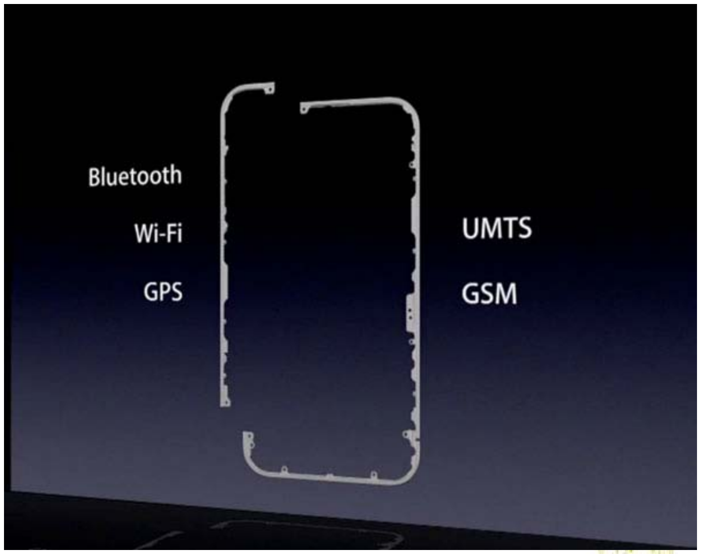
Why Study EM?

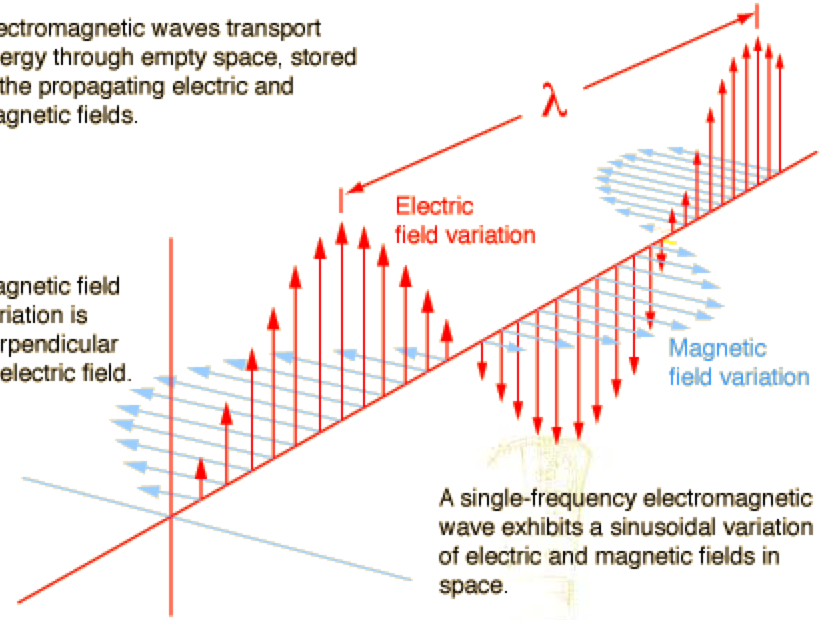
- Because EM phenomena is in all electrical/electronic based equipment ….
- Electrical/electronic components/equipment are almost everywhere …
- Thus, EM is everywhere these days …
- Computers, Cell Phones, Car Controllers, Power Lines, etc.
- Wireless communications is based on EM wave propagation …
- High speed digital design is based on EM wave propagation …
- Fields around power lines are EM fields …



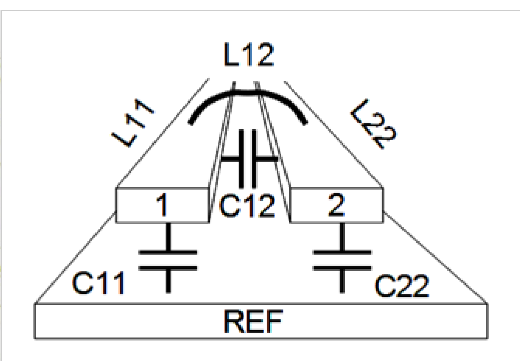
Receiver
f_1 = 6 GHz
M_1
Mixer
f_0 = 2 GHz
Oscillator
M_2
f_2 = 4 GHz
C_2
C_3
Circulator
C_1
Transmitter
f_2 = 4 GHz
Satellite segment
6 GHz
4 GHz
Uplink @ 6 GHz
Downlink @ 4 GHz
Ground station 1
Ground station 2
Transmitter
Receiver
Coder
coax
Multiplexer
coax
Frequency translator
Voice (Telephone)
optical fiber
Video and voice
coax
Receiver
coax
Transmitter
Coder
Data
Terrestrial microwave link

Figure 1.1


Bluetooth
Wi-Fi
GPS
UMTS
GSM


Electromagnetic waves transport energy through empty space, stored in the propagating electric and magnetic fields.
λ
Electric field variation
Magnetic field variation is perpendicular to electric field.
Magnetic field variation
A single-frequency electromagnetic wave exhibits a sinusoidal variation of electric and magnetic fields in space.

?


L12
L11
L22
1
C12
2
C11
C22
REF

Vector Algebra Quick Review (MATH302!)

- Chapters 1, 2 and 3 in your text book!

$$\mathbf{A} = A\hat{a}_A = A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z \quad \text{(cartesian)}$$

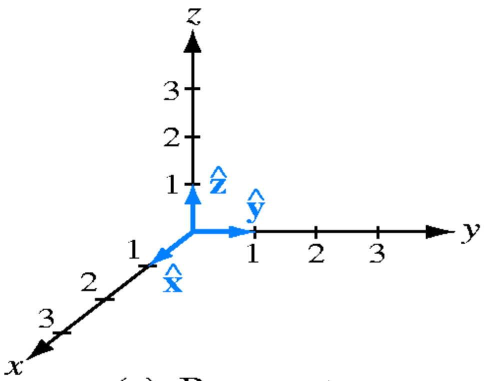
$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$\hat{a}_A = \frac{\mathbf{A}}{|\mathbf{A}|} = \frac{\mathbf{A}}{A} = \frac{A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

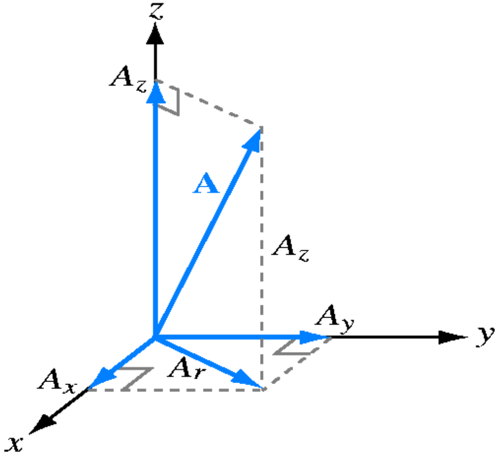
projection of a vector on an axis or another vector,

$$A_x = \mathbf{A} \bullet \hat{a}_x \quad , \quad A_y = \mathbf{A} \bullet \hat{a}_y \quad , \quad A_z = \mathbf{A} \bullet \hat{a}_z$$

$$\mathbf{A} \bullet \mathbf{B} = AB\cos\theta_{AB} = A_xB_x + A_yB_y + A_zB_z$$



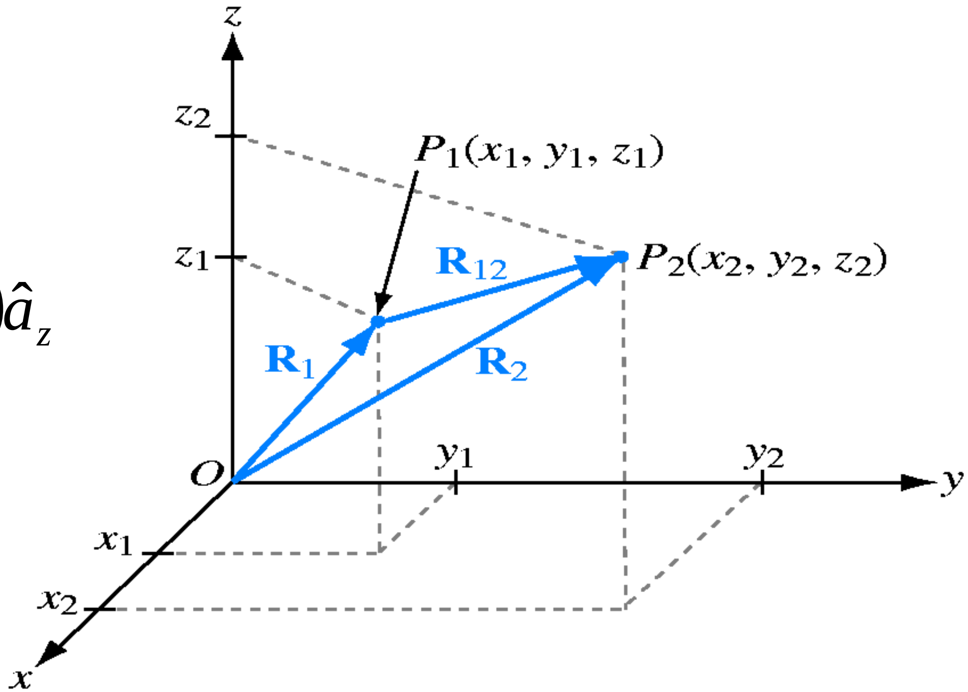
(a) Base vectors



(b) Components of **A**

- Position Vector,

$$R_{12} = R_2 - R_1$$
$$= (x_2 - x_1)\hat{a}_x + (y_2 - y_1)\hat{a}_y + (z_2 - z_1)\hat{a}_z$$



- **Example 1.1:**

Given a vector

$$\mathbf{A} = -\hat{a}_x + 2\hat{a}_y - 2\hat{a}_z$$

find the magnitude of A, its unit vector and the angle that it makes with the z-axis? [On the Board.]

- Cross Product,

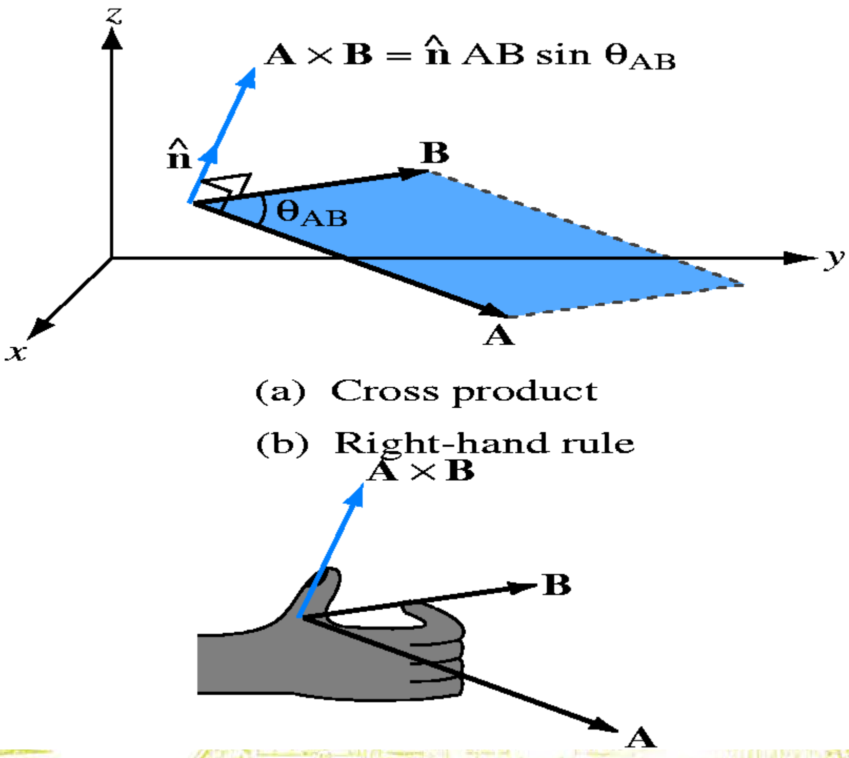
$$\mathbf{A}\times\mathbf{B} = AB\sin\theta_{AB}\hat{a}_n = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\hat{a}_x \times \hat{a}_y = \hat{a}_z$$

$$\hat{a}_y \times \hat{a}_z = \hat{a}_x$$

$$\hat{a}_z \times \hat{a}_x = \hat{a}_y$$

$$\mathbf{A}\times\mathbf{B} = -\mathbf{B}\times\mathbf{A}$$



- Scalar Triple product,

$$\mathbf{A}\bullet(\mathbf{B}\times\mathbf{C}) = \mathbf{B}\bullet(\mathbf{C}\times\mathbf{A}) = \mathbf{C}\bullet(\mathbf{A}\times\mathbf{B})$$

$$= \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

- Vector Triple Product,

$$\mathbf{A}\times(\mathbf{B}\times\mathbf{C})=\mathbf{B}\bullet(\mathbf{C}\bullet\mathbf{A})-\mathbf{C}\bullet(\mathbf{A}\bullet\mathbf{B})$$

A) Cartesian Coordinates (x,y,z)

- already dealt with!

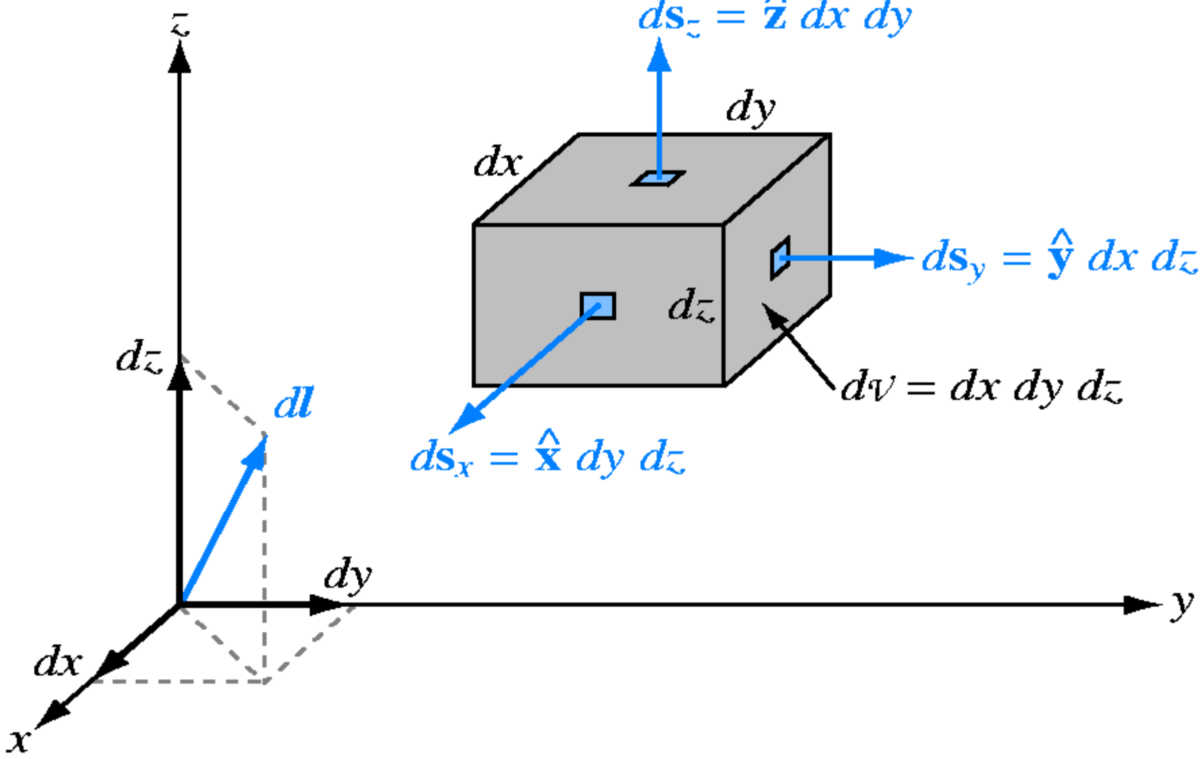
$$d\mathbf{l} = dx\hat{a}_x + dy\hat{a}_y + dz\hat{a}_z$$

$$d\mathbf{s} = dydz\hat{a}_x$$

$$= dxdz\hat{a}_y$$

$$= dxdy\hat{a}_z$$

$$dv = dxdydz$$



B) Cylindrical Coordinates (ρ,ϕ,z)

- useful for problems with cylindrical symmetry

$$\mathbf{A} = A_\rho \hat{a}_\rho + A_\phi \hat{a}_\phi + A_z \hat{a}_z$$

$$A = \sqrt{A_\rho^2 + A_\phi^2 + A_z^2}$$

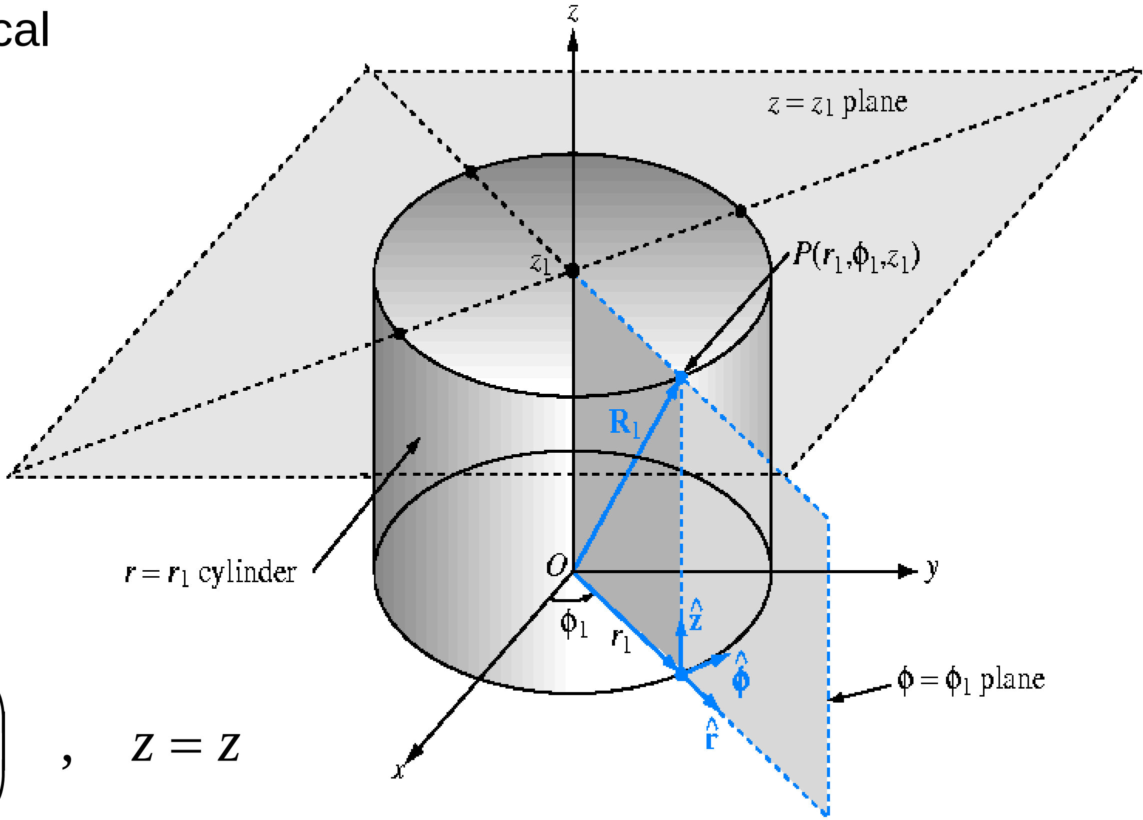
$$\hat{a}_\rho \times \hat{a}_\phi = \hat{a}_z$$

$$\hat{a}_\phi \times \hat{a}_z = \hat{a}_\rho$$

$$\hat{a}_z \times \hat{a}_\rho = \hat{a}_\phi$$

$$\rho = \sqrt{x^2 + y^2} \quad , \quad \phi = \tan^{-1}\left(\frac{y}{x}\right) \quad , \quad z = z$$

$$x = \rho\cos\phi \quad , \quad y = \rho\sin\phi \quad , \quad z = z$$



- Transformation matrices

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\varphi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\varphi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

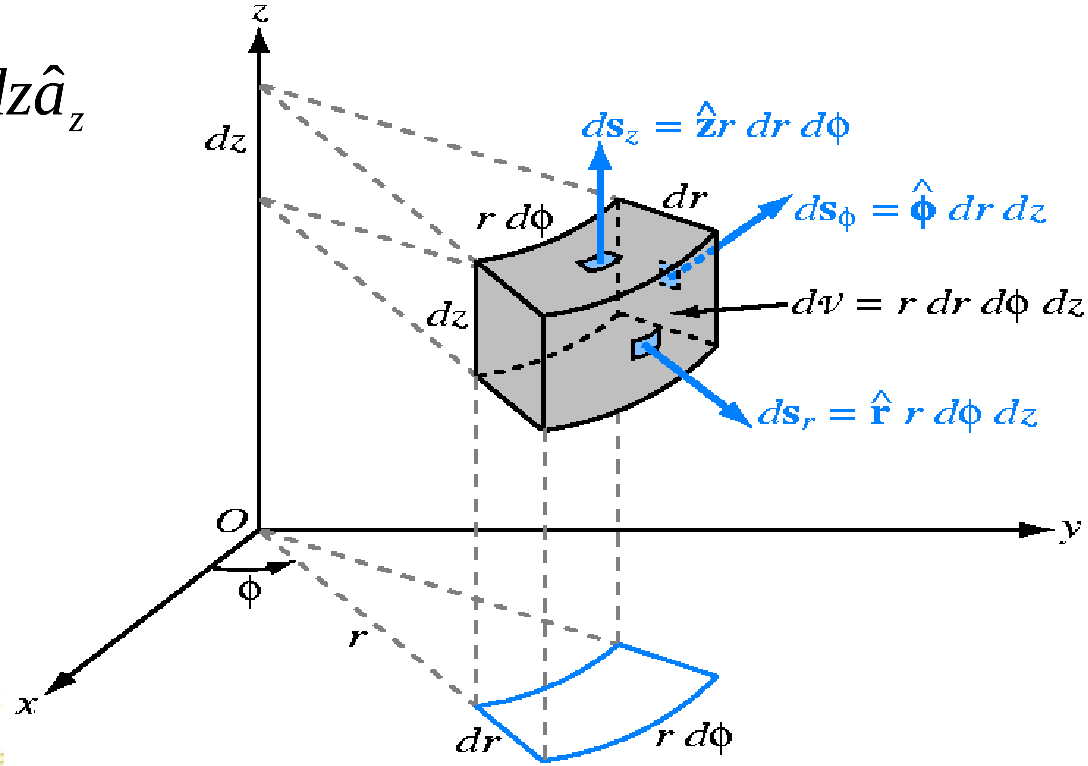
- $$d\mathbf{l} = d\rho\hat{a}_\rho + \rho d\phi\hat{a}_\phi + dz\hat{a}_z$$

$$d\mathbf{s} = \rho d\phi dz\hat{a}_\rho$$

$$= d\rho dz\hat{a}_\phi$$

$$= \rho d\rho d\phi\hat{a}_z$$

$$dv = \rho d\rho d\phi dz$$



C) Spherical Coordinate System (r,θ,φ)

- used in problems with spherical symmetry.

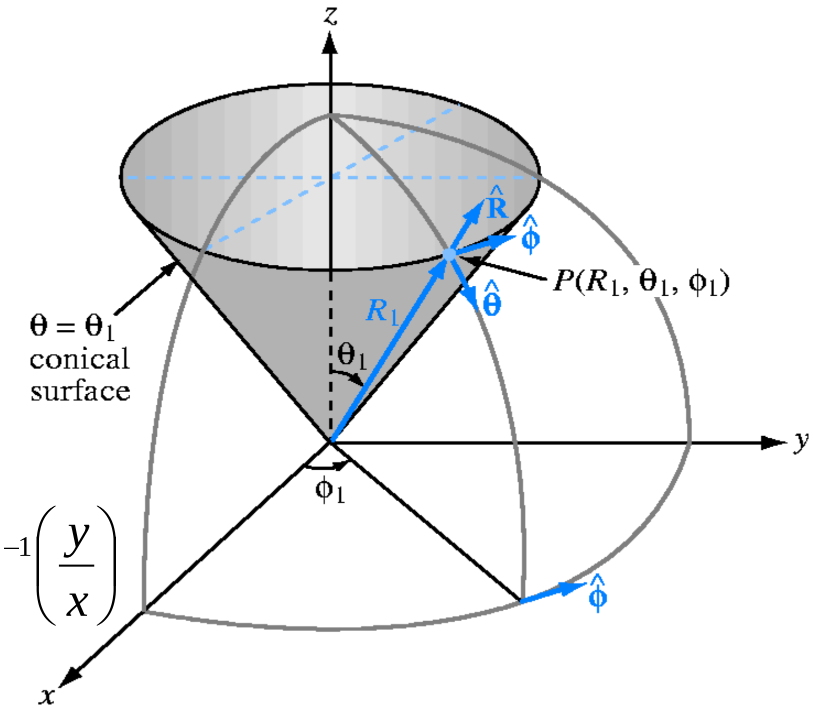
$$\mathbf{A} = A_r\hat{a}_r + A_\theta\hat{a}_\theta + A_\phi\hat{a}_\phi$$

$$A = \sqrt{A_\rho^2 + A_\theta^2 + A_\phi^2}$$

$$\hat{a}_r \times \hat{a}_\theta = \hat{a}_\phi$$

$$\hat{a}_\theta \times \hat{a}_\phi = \hat{a}_r$$

$$\hat{a}_\phi \times \hat{a}_r = \hat{a}_\theta$$



$$r = \sqrt{x^2 + y^2 + z^2} \quad , \quad \theta = \tan^{-1}\left(\frac{\sqrt{x^2 + y^2}}{z}\right) \quad , \quad \phi = \tan^{-1}\left(\frac{y}{x}\right)$$

$$x = r\sin\theta\cos\phi \quad , \quad y = r\sin\theta\sin\phi \quad , \quad z = r\cos\theta$$

- Transformation Matrices,

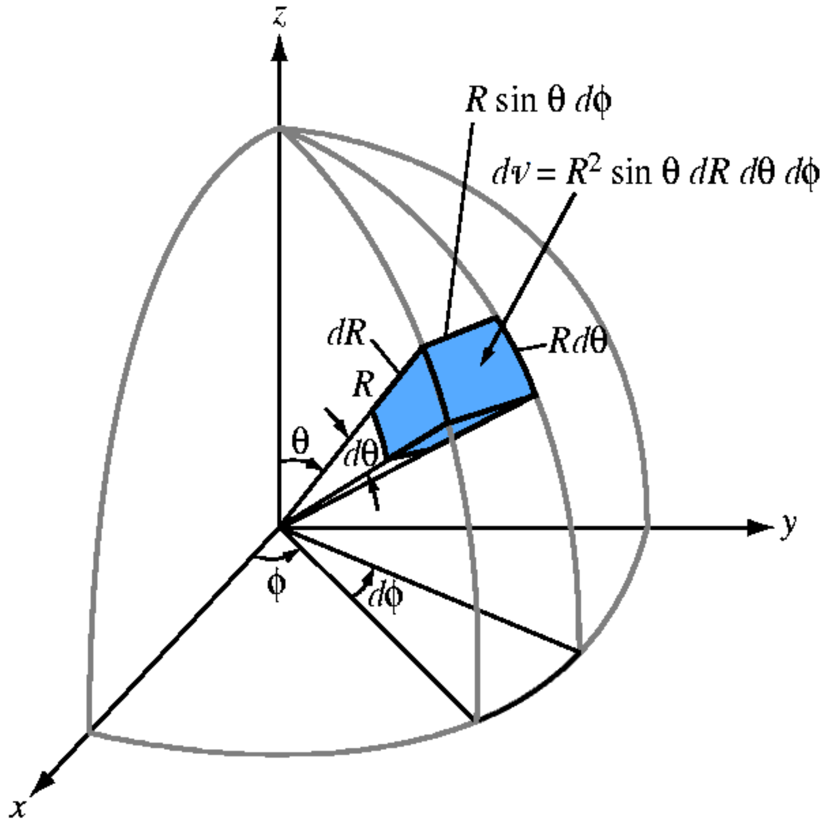
$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\phi & \sin\theta\sin\varphi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\varphi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

- $d\mathbf{l} = dr\hat{a}_r + rd\theta\hat{a}_\theta + r\sin\theta d\theta d\phi\hat{a}_\phi$

$$d\mathbf{s} = r^2 \sin\theta d\theta d\phi \hat{a}_r$$
$$= r\sin\theta dr d\phi \hat{a}_\theta$$
$$= r dr d\phi \hat{a}_\phi$$

$$dv = r^2 \sin\theta dr d\theta d\phi$$

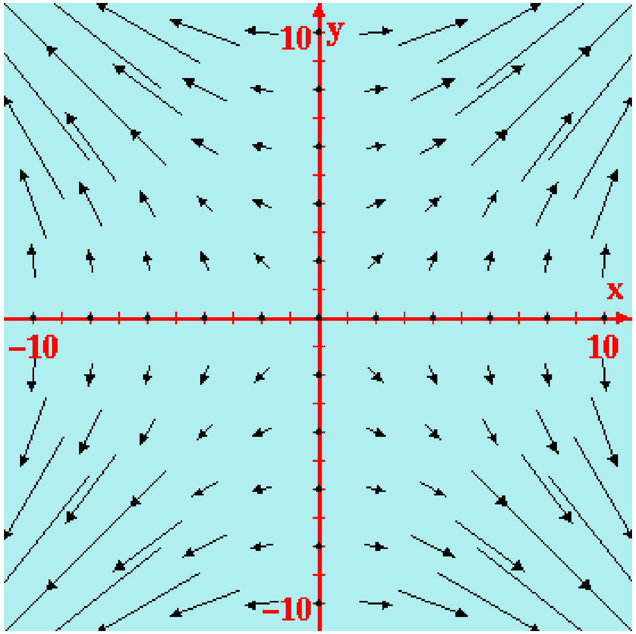


Gradient of a Scalar Function

- A vector (magnitude and direction) that represents the maximum space rate of increase of a function *A*.

$$\nabla A = \frac{\partial A}{\partial x}\hat{a}_x + \frac{\partial A}{\partial y}\hat{a}_y + \frac{\partial A}{\partial z}\hat{a}_z$$

- Gradient in other coordinate systems is given in your text book.



$$T = x^2 y^2$$

$$\nabla T = \hat{\mathbf{x}}\frac{\partial T}{\partial x} + \hat{\mathbf{y}}\frac{\partial T}{\partial y} + \hat{\mathbf{z}}\frac{\partial T}{\partial z}$$

$$= \hat{\mathbf{x}}\,2xy^2 + \hat{\mathbf{y}}\,2x^2 y$$

Divergence of a Vector and Divergence Theorem

- The divergence of A at a point P is the outward flux per unit volume as the volume shrinks about P.

$$\nabla \bullet A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

- The divergence theorem states that the total outward flux of a vector field *A* through the closed surface *S* is the same as the volume integral of the divergence of *A*.

$$\oint_S \vec{A} \bullet d\vec{s} = \int_V \nabla \bullet \vec{A} dv$$

Curl of a Vector and Stokes's Theorem

- The curl of A is a rotational vector whose magnitude is the maximum circulation of A per unit area as the area tends to zero and whose direction is normal to the direction of the area when oriented to make maximum circulation.

$$curl\left(A\right) = \nabla \times A$$

- Stokes's Theorem: The circulation of a vector field A around a closed path is equal to the surface integral of the curl of A over the open surface bounded by the path.

$$\oint_L \vec{A} \bullet d\vec{l} = \int_S \left(\nabla \times \vec{A}\right) \bullet d\vec{s}$$

- More exercises in the book and during the first few lab sessions.

Next time ...

- Do Not Forget to check the class page often …