Schelkunoff's polynomial method

$$AF = \sum_{n=1}^{N} a_n e^{j(n-1)(kd\cos\theta+\beta)} = \sum_{n=1}^{N} a_n e^{j(n-1)\psi}$$

$$z = x + jy = e^{j\psi} = e^{j(kd\cos\theta+\beta)}$$

$$\therefore AF = \sum_{n=1}^{N} a_n z^{n-1} = a_1 + a_2 z + a_3 z^2 + \dots a_N z^{N-1}$$

$$= a_N (z - z_1)(z - z_2) \dots (z - z_{N-1})$$

$z_1, \dots z_{N-1}$ are the roots of the polynomial

$$|AF| = |a_N|\,|z-z_1|\,|z-z_2| \dots |z-z_{N-1}|$$

$$\rightarrow \quad z = |z| e^{j\psi} = |z| \angle\psi = 1\angle\psi$$

where $\psi = kd\cos\theta + \beta = \frac{2\pi}{\lambda} d\cos\theta + \beta$

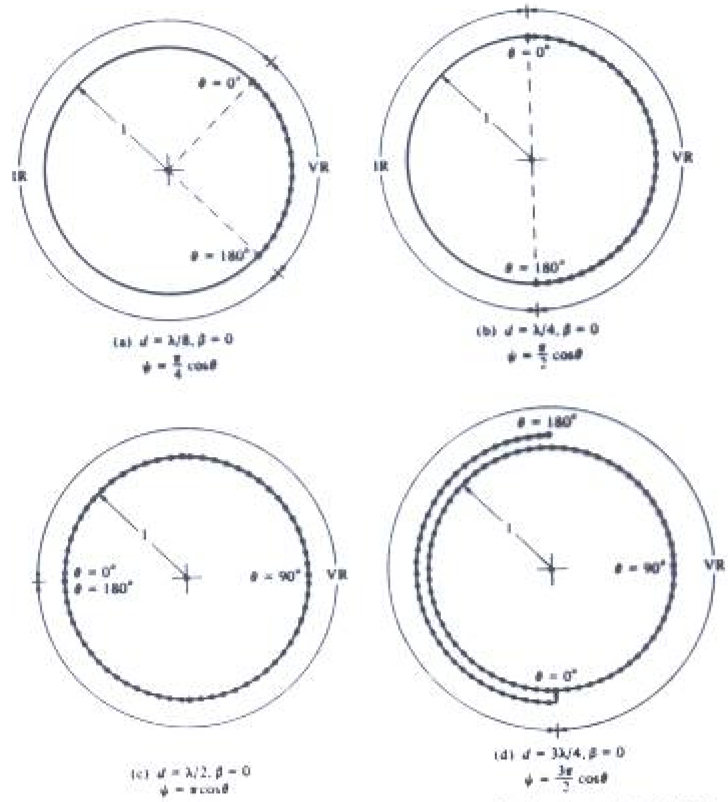
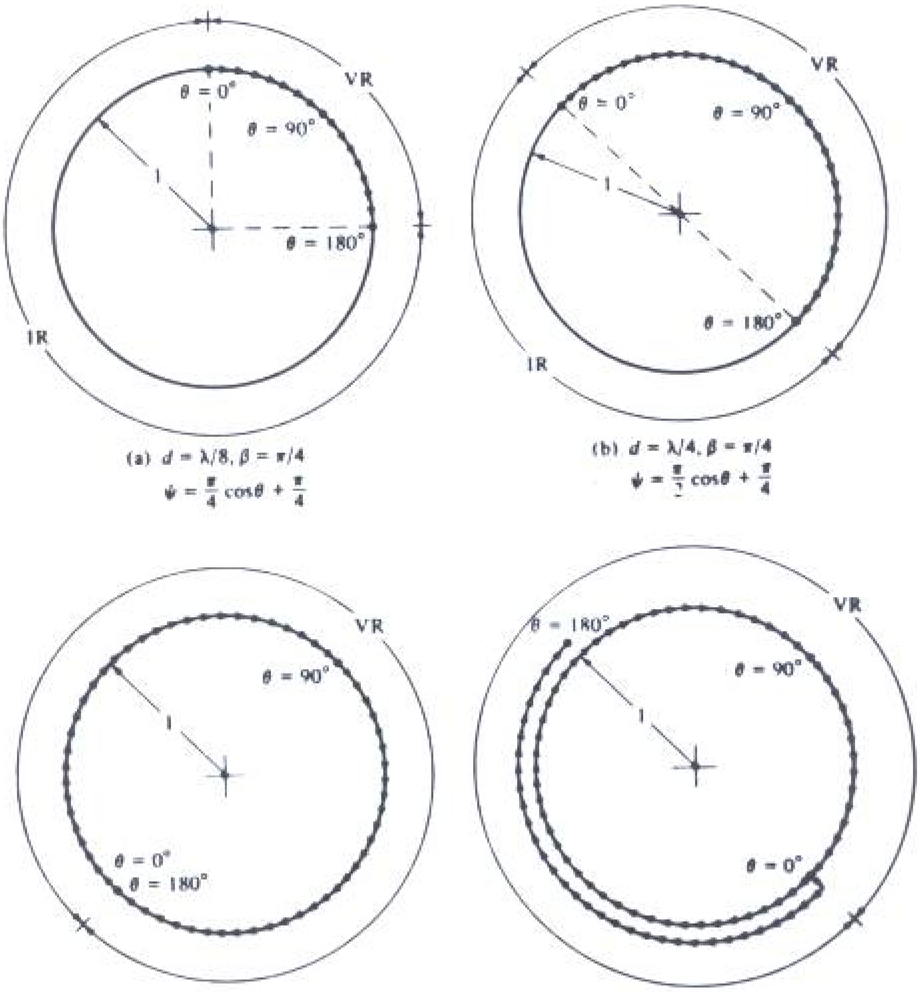


Figure 7.2 *Visible Region* (VR) and *Invisible Region* (IR) boundaries for complex variable z when $\beta = 0$

$= \frac{\pi}{4}$



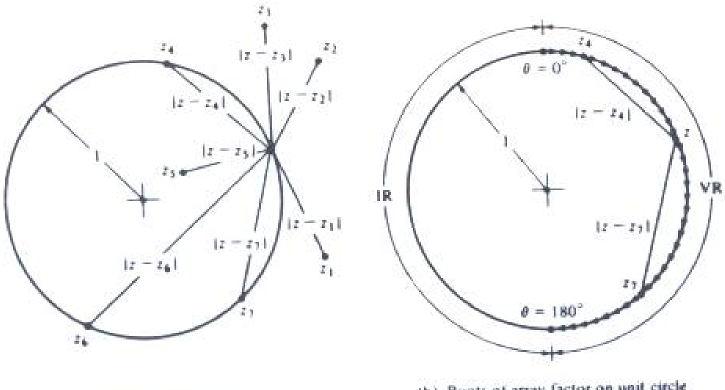
(a) $d = \lambda/8$, $\beta = \pi/4$
$\psi = \frac{\pi}{4}\cos\theta + \frac{\pi}{4}$

(b) $d = \lambda/4$, $\beta = \pi/4$
$\psi = \frac{\pi}{2}\cos\theta + \frac{\pi}{4}$

(c) $d = \lambda/2$, $\beta = \pi/4$
$\psi = \pi\cos\theta + \frac{\pi}{4}$

(d) $d = 3\lambda/4$, $\beta = \pi/4$
$\psi = \frac{3\pi}{2}\cos\theta + \frac{\pi}{4}$

Figure 7.3 *V*isible *R*egion (VR) and *I*nvisible *R*egion (IR) boundaries for complex variable z when $\beta = \pi/4$.



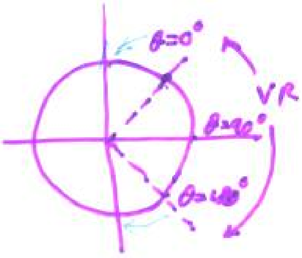
(a) Roots of array factor

(b) Roots of array factor on unit circle and within visible region

Figure 7.4 Array factor roots within and outside unit circle, and visible and invisible regions.

Example

Design a linear array with $d = \frac{\lambda}{4}$, such that it has zeros at $\theta = 0°, 90°,$ and $180°$. Determine No. of elements, their excitation & plot the array factor.



The three nulls correspond to

$$z = j,\ 1,\ -j$$

$$\therefore AF = (z - z_1)(z - z_2)(z - z_3)$$
$$= (z - j)(z - 1)(z + j)$$
$$= z^3 - z^2 + z - 1$$

∴ array excitations are $a_1 = -1,\ a_2 = 1,\ a_3 = -1,\ a_4 = 1$