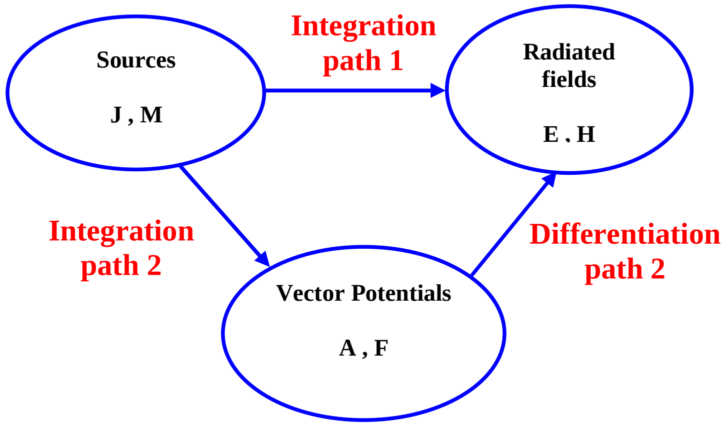
RADIATION INTEGRALS & POENTIAL FUNCTIONS



Vector Potential A for electric current source J

$$\nabla \bullet B = 0$$

$$B_A = \mu H_A = \nabla \times A \quad \rightarrow \quad or \;\; H_A = \frac{1}{\mu} \nabla \times A$$

$$\nabla^2 A + k^2 A = -\mu J$$

$$E_A = -j\omega A - \frac{j}{\omega\mu\varepsilon} \nabla(\nabla \bullet A)$$

Once we find A, we can find E_A and H_A everywhere.

Vector Potential F for magnetic current source M

If $J = 0 \quad and \quad M \neq 0 \quad \rightarrow \quad \nabla \bullet D = 0$

$$E_F = -\frac{1}{\varepsilon} \nabla \times F$$

$$and \quad H_F = -j\omega F - \frac{j}{\omega\mu\varepsilon} \nabla(\nabla \bullet F)$$

$$\nabla^2 F + k^2 F = -\varepsilon M$$

Once we find F, we can find H_F and E_F everywhere.

Solution of the inhomogeneous vector potential wave equation

The time varying solution of $\nabla^2 A + k^2 A = -\mu J$ **is:**

$$A = \frac{\mu}{4\pi} \iiint_v J \frac{e^{-jkr}}{r} dv'$$

If the source is not at the origin, but at (x', y', z') then the solution can be written as:

$$A(x,y,z) = \frac{\mu}{4\pi} \iiint_v J(x',y',z') \frac{e^{-jkR}}{R} dv'$$

Similarly, the solution of the electric vector potential is:

$$F(x,y,z) = \frac{\varepsilon}{4\pi} \iiint_v M(x',y',z') \frac{e^{-jkR}}{R} dv'$$

If J and M represent surface densities, then the solutions become:

$$A = \frac{\mu}{4\pi} \iint_s J_s \frac{e^{-jkR}}{R} ds'$$

and $$F = \frac{\varepsilon}{4\pi} \iint_s M_s \frac{e^{-jkR}}{R} ds'$$

For linear densities I_e and I_m:

$$A = \frac{\mu}{4\pi} \int_c I_e \frac{e^{-jkR}}{R} dl'$$

and $$F = \frac{\varepsilon}{4\pi} \int_c I_M \frac{e^{-jkR}}{R} dl'$$

Procedure to find E and H fields due to J and M current densities

1. **Specify J and M (electric and magnetic current densities).**
2. **Find A due to J and F due to M from:**

$$A(x,y,z) = \frac{\mu}{4\pi} \iiint_v J(x',y',z') \frac{e^{-jkR}}{R} dv',$$

$$F(x,y,z) = \frac{\varepsilon}{4\pi} \iiint_v M(x',y',z') \frac{e^{-jkR}}{R} dv'.$$

3. **Find H_A and E_A due to J from:**

$$H_A = \frac{1}{\mu} \nabla \times A \quad , \quad E_A = -j\omega A - \frac{j}{\omega\mu\varepsilon} \nabla(\nabla \bullet A)$$

E_A may also be obtained from $\nabla \times H_A = J + j\omega\varepsilon E_A$

By setting *J* = 0 in the field region.

4. **Find E_F and H_F due to *M* from:**

$$E_F = -\frac{1}{\varepsilon}\nabla \times F$$

$$and \quad H_F = -j\omega F - \frac{j}{\omega\mu\varepsilon}\nabla(\nabla \bullet F)$$

H_F may also be obtained from $\nabla \times E_F = -M - j\omega\mu H_F$
By setting M = 0 in the field region.

5. **The total radiated fields are then given by:**

$$E = E_A + E_F$$

$$H = H_A + H_F$$