Home Work #3

4.15 $\bar{A} = \hat{a}_z \frac{\mu I_o}{4\pi} \int_0^{\ell} e^{-jkz'} \frac{e^{-jkR}}{R} dz' \simeq \hat{a}_z \frac{\mu I_o}{4\pi r} e^{-jkr} \int_0^{\ell} e^{-jk(1-\cos\theta)z'} dz'$

using the substitution $u = -jk(1-\cos\theta)z' \rightarrow \therefore du = -jk(1-\cos\theta)dz'$

$\therefore \bar{A} = \hat{a}_z \frac{\mu I_o}{4\pi r} e^{-jkr} \int_0^{[-jk\ell(1-\cos\theta)]} \frac{e^u du}{-jk(1-\cos\theta)} = \hat{a}_z \frac{\mu I_o}{4\pi r} e^{-jkr} \frac{e^{-jk\ell(1-\cos\theta)} - 1}{-jk(1-\cos\theta)}$

or $A_z = \frac{\mu I_o \ell}{4\pi r} e^{-jkr} e^{-j\frac{k\ell}{2}(1-\cos\theta)} \frac{\sin \frac{k\ell}{2}(1-\cos\theta)}{\frac{k\ell}{2}(1-\cos\theta)}$

or using $F = \frac{k\ell}{2}(1-\cos\theta)$

$\therefore A_z = \frac{\mu I_o \ell}{4\pi r} e^{-jkr} e^{-jF} \frac{\sin F}{F}$

(a) Using $\left\{ \begin{array}{l} A_r = A_z \cos\theta \\ A_\theta = -A_z \sin\theta \\ A_\phi = 0 \end{array} \right. \longrightarrow \therefore$ far field $\left\{ \begin{array}{l} E_\theta \simeq -j\omega A_\theta \\ E_\phi \simeq -j\omega A_\phi \\ E_r \simeq 0 \end{array} \right.$

$\therefore E_\theta = j\omega A_z \sin\theta = j \frac{\omega \mu I_o \ell}{4\pi r} e^{-jkr} e^{-jF} \frac{\sin F}{F} \sin\theta$

$H_\phi \simeq \frac{E_\theta}{\eta}$

$E_r = H_r = E_\phi = H_\theta = 0$

(b) $\bar{W}_{rad} = \frac{1}{2} Re [E \times H^*] = \frac{1}{2} E_\theta H_\phi^* = \frac{1}{2\eta} |E_\theta|^2$

$= \frac{1}{2\eta} \left| \frac{\omega \mu I_o \ell}{4\pi r} \frac{\sin F}{F} \sin\theta \right|^2$

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4.19 $\rho = \frac{1+|\Gamma|}{1-|\Gamma|}$, $\Gamma = \frac{R_{in} - Z_o}{R_{in} + Z_o}$, $R_{in} = \frac{R_r}{\sin^2(\frac{k\ell}{2})}$, $Z_o = 50\,\Omega$

(a) $\ell = \lambda/4$, $\frac{k\ell}{2} = \pi/4$, $k\ell = \pi/2$, $2k\ell = \pi$

$\therefore R_r = 60 \{ C + \ln(\pi/2) - C_i(\pi/2) + \frac{1}{2} \sin(\frac{\pi}{2}) [S_i(\pi) - 2 S_i(\pi/2)] \}$

$= 60 \{ 0.5772 + 0.45158 - 0.47 + \frac{1}{2} [1.85 - 2(1.3698)] \} = 6.8388\,\Omega$

$\therefore R_{in} = \frac{6.8388}{\sin^2(\frac{\pi}{4})} = 13.6776\,\Omega$

$\Gamma = \frac{13.6776 - 50}{13.6776 + 50} = -0.5704 \rightarrow \rho = \frac{1 + 0.5704}{1 - 0.5704} = 3.6555$

(b) $\ell = \lambda/2$, $\frac{k\ell}{2} = \pi/2$, $k\ell = \pi$, $2k\ell = 2\pi$

$R_r = 60 \{ C + \ln(\pi) - C_i(\pi) + \frac{1}{2} \cos(\pi) [C + \ln(\frac{\pi}{2}) + C_i(2\pi) - 2C_i(\pi)] \}$

$= 73.13\,\Omega$

$$R_{in} = \frac{73.13}{\sin^2(\pi/2)} = 73.13\ \Omega$$

$$\Gamma = \frac{73.13-50}{73.13+50} = 0.1879 \rightarrow \rho = \frac{1+0.1879}{1-0.1879} = 1.4626$$

(c) $\ell = 3\lambda/4$, $k\ell/2 = 3\pi/4$, $k\ell = 3\pi/2$, $2k\ell = 3\pi$

$$\therefore R_r = 60\left\{0.5772 + \ln(3\pi/2) - C_i(\tfrac{3\pi}{2}) + \tfrac{1}{2}\sin(\tfrac{3\pi}{2})\left[S_i(3\pi) - 2S_i(\tfrac{3\pi}{2})\right]\right\}$$

$$= 185.965\ \Omega$$

$$R_{in} = 185.965/\sin^2(3\pi/4) = 371.93$$

$$\therefore \Gamma = 0.763,\quad \rho = 7.4386$$

(d) $\ell = \lambda$, $k\ell/2 = \pi$, $k\ell = 2\pi$, $2k\ell = 4\pi$

$$R_r = 60\left\{0.5772 + \ln(2\pi) - C_i(2\pi) + \tfrac{1}{2}\cos(2\pi)\left[0.5772 + \ln(\pi) + C_i(4\pi) - 2C_i(2\pi)\right]\right\}$$

$$= 199.1\ \Omega \longrightarrow R_{in} = \frac{199.1}{\sin^2(\pi)} = \infty$$

$$\therefore \Gamma = \frac{\infty - 50}{\infty + 50} = 1 \longrightarrow \rho = \infty$$

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4.20 (a) $R_r = 80\pi^2(\frac{\ell}{\lambda})^2$, $a = 10^{-4}\lambda$, $f = 10$ MHZ, $\sigma = 5.7\times10^7$ S/m

$$R_L = R_{hf} = \frac{\ell}{P}\sqrt{\frac{\omega\mu}{2\sigma}} = \frac{\ell}{2\pi a}\sqrt{\frac{\omega\mu_0}{2\sigma}} = \frac{\ell}{2\pi\times10^{-4}\lambda}\sqrt{\frac{2\pi10^7(4\pi10^{-7})}{2(5.7\times10^7)}}$$

$$= 1.3245\left(\frac{\ell}{\lambda}\right), \quad e_{cd} = \frac{R_r}{R_L+R_r}$$

$$\therefore \text{fr } \ell = \frac{\lambda}{50} \rightarrow R_r = 80\pi^2(\tfrac{1}{50})^2 = 0.316\ \Omega$$

$$R_L = R_{hf} = 1.3245(\tfrac{1}{50}) = 0.02649\ \Omega$$

$$\therefore e_{cd} = \frac{0.316}{0.316+0.02649}\times100 = 92.26\ \%$$

(b) $\ell = \lambda/4$, from prob. 4.19 $R_r = 6.8388\ \Omega$

$$R_L = R_{hf} = \frac{1.3245}{4} = 0.3311 \rightarrow e_{cd} = \frac{6.8388\times100}{6.8388+0.3311} = 95.38\ \%$$

(c) $\ell = \lambda/2$ $\quad R_r = 73.13\ \Omega$, $R_L = \frac{1.3245}{2} = 0.66225\ \Omega$

$$\therefore e_{cd} = 99.1\ \%$$

(d) $\ell = \lambda$, $R_r = 199.1\ \Omega$, $R_L = 1.3245\ \Omega$

$$\therefore e_{cd} = \frac{199.1\times100}{199.1+1.3245} = 99.34\ \%$$



(2/3)

4-33) $\frac{kl}{2} = \frac{3\pi}{4}, \quad kl = \frac{3\pi}{2}, \quad and \quad 2kl = 3\pi$

a. Using eqn.(4.70) we get $R_r = 185.808\ \Omega$

b. $R_{in} = \frac{R_r}{\sin^2(3\pi/4)} = 371.617\ \Omega$

c.
$$\Gamma = \frac{371.617 - 300}{371.617 + 300} = 0.10663$$
$$VSWR = \frac{1 + 0.10663}{1 - 0.10663} = 1.2387$$

Computer Program to compute the dipole radiation power, directivity, radiation resistance and plots the radiation pattern is listed below.

```
% © Prof. Mahmoud M. Dawoud
m=input('dipole length?');l=m(1,1);
kl=2*pi*l;global a;
a=kl./2;
theta=0.000000000001:1:180.000000000001;
thetar=theta.*pi./180;
ftheta=(cos(a.*cos(thetar))-cos(a))./sin(thetar);
ftheta=ftheta.*ftheta;
fmax=max(ftheta');ftheta=ftheta./fmax;
qq=quad8('q',0.0000001,pi);prad=30.*qq;
Rr=2.*prad;
D0=2.0.*fmax./qq;
Results=[prad Rr D0]
plot(theta',ftheta');grid;
```