Radiated fields of a finite length dipole

We are interested in the far field.

Finite length dipole is subdivided in a large number of infinitesimal dipoles of length dz'.

The fields of the infinitesimal dipole are:

$$dE_\theta \simeq j\eta \frac{k I_e(x',y',z') e^{-jkR}}{4\pi R} \sin\theta \, dz'$$

$$dH_\phi \simeq j \frac{k I_e(x',y',z') e^{-jkR}}{4\pi R} \sin\theta \, dz'$$

& $dE_r \simeq dE_\phi = dH_r = dH_\theta = 0$

Using far field approximation

$R \simeq r$ for amplitude

$R \simeq r - z'\cos\theta$ for phase

$$\therefore \; dE_\theta \simeq j\eta \frac{k I_e(x',y',z')}{4\pi r} e^{-jkr} \sin\theta \, e^{jkz'\cos\theta} \, dz'$$

$$\& \; E_\theta = \int_{-l/2}^{l/2} dE_\theta = j\eta \frac{k e^{-jkr}}{4\pi r} \sin\theta \left[\int_{-l/2}^{l/2} I_e(x',y',z') e^{jkz'\cos\theta} dz' \right]$$

Total Field = (Element factor) x (space factor)

$$E_\theta = j\eta \frac{k e^{-jkr}}{4\pi r} \sin\theta \left[\int_{-l/2}^{l/2} I_e \, e^{jkz'\cos\theta} dz' \right]$$

$$E_\theta \simeq j\eta \frac{k I_0 e^{-jkr}}{4\pi r} \sin\theta \left[\int_{-l/2}^{0} \sin\left[k\left(\frac{l}{2}+z'\right)\right] e^{jkz'\cos\theta} dz' + \int_{0}^{l/2} \sin\left[k\left(\frac{l}{2}-z'\right)\right] e^{jkz'\cos\theta} dz' \right]$$

using $\int e^{\alpha x} \sin(\beta x+\gamma)\, dx$

$$= \frac{e^{\alpha x}}{\alpha^2+\beta^2} \left[\alpha \sin(\beta x+\gamma) - \beta \cos(\beta x+\gamma) \right]$$

where $\alpha = jk\cos\theta$

$\beta = \pm k$

$\gamma = kl/2$

$$\therefore E_\theta \simeq j\eta \frac{I_0 e^{-jkr}}{2\pi r} \left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta} \right]$$

$$\& \ H_\phi \simeq j \frac{I_0 e^{-jkr}}{2\pi r} \left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta} \right]$$

Power density:

$$\bar{W}_{av} = \hat{a}_r W_{av} = \hat{a}_r \frac{1}{2\eta} |E_\theta|^2$$

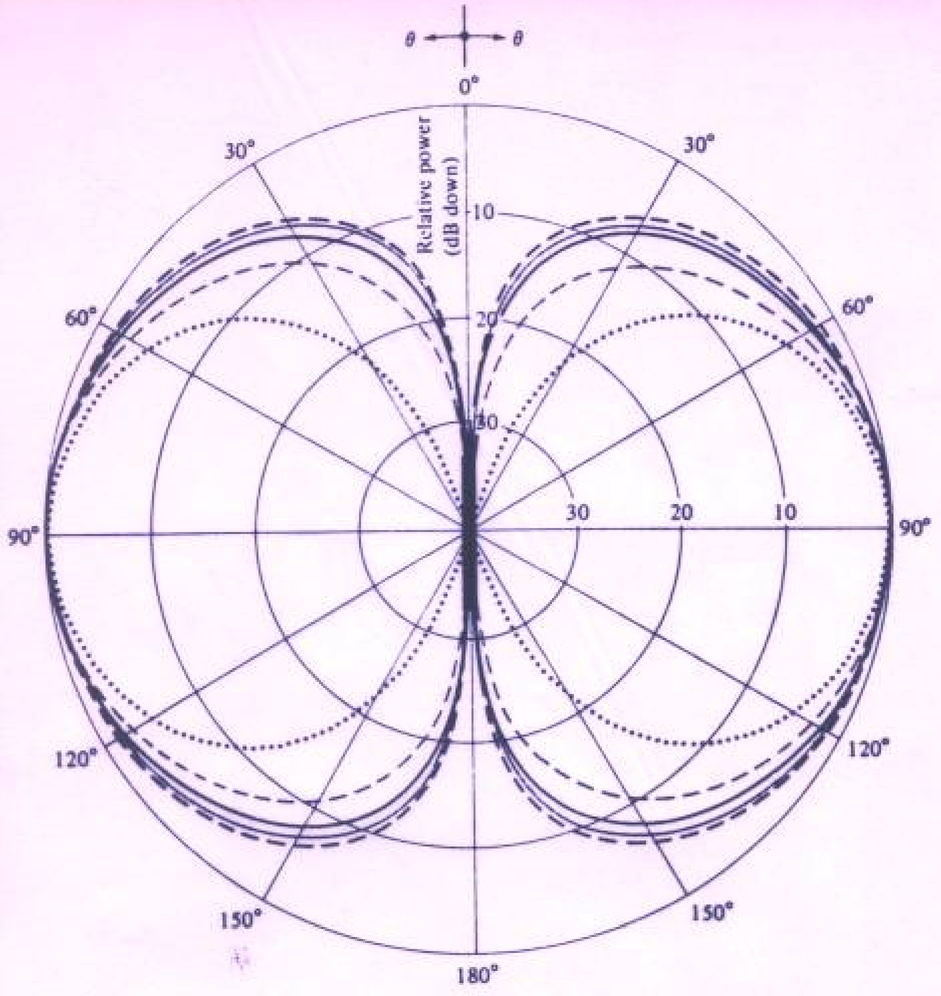
$$\therefore \; W_{av} = \eta \frac{|I_0|^2}{8\pi^2 r^2} \left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta} \right]^2$$

Radiation intensity:

$$U = r^2 W_{av} = \eta \frac{|I_0|^2}{8\pi^2} \left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta} \right]^2$$

From the elevation plane amplitude patterns, we can find HPBW (3-dB beamwidth)

	HPBW
$l \ll \lambda$	90°
$l = \lambda/4$	87°
$l = \lambda/2$	78°
$l = 3\lambda/4$	64°
$l = \lambda$	47.8°



- - - - - $l << \lambda$

——— $l = \lambda/4$

——— $l = \lambda/2$

- - - - - $l = 3\lambda/4$

.... $l = \lambda$

Figure 4.5 Elevation plane amplitude patterns for a thin dipole with sinusoidal current

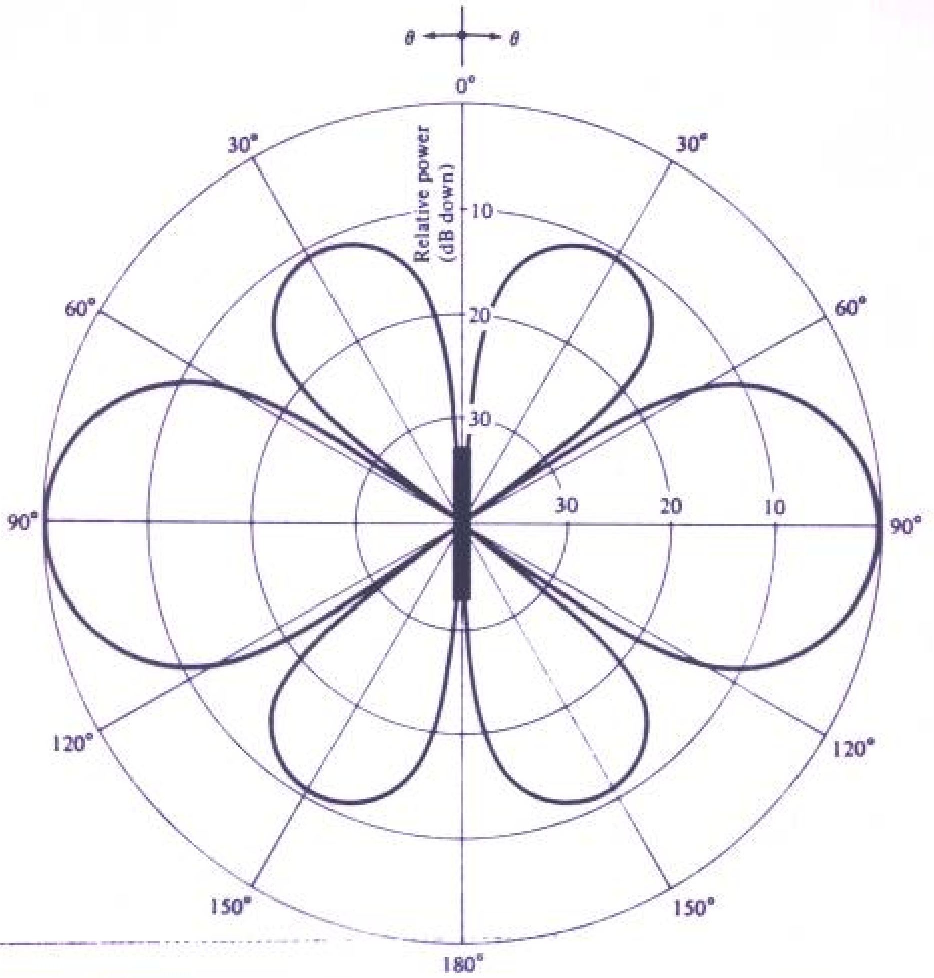


Figure 4.6 Elevation plane amplitude pattern for a thin dipole of $l = 1.25\lambda$ and sinusoidal current distribution.

Total radiated power :

$$P_{rad} = \oiint_S W_{av} \cdot ds = \iint U \sin\theta \, d\theta \, d\phi$$

$$= \int_0^{2\pi}\int_0^{\pi} \eta \frac{|I_0|^2}{8\pi^2} \left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta} \right]^2 \sin\theta \, d\theta \, d\phi$$

$$= \eta \frac{|I_0|^2}{4\pi} \int_0^{\pi} \frac{\left[\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)\right]^2}{\sin\theta} d\theta$$

$$= \eta \frac{|I_0|^2}{4\pi} \cdot Q$$

where $$Q = \int_0^{\pi} \frac{\left[\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)\right]^2}{\sin\theta} d\theta$$

$$= C + \ln(kl) - C_i(kl)$$

$$+ \frac{1}{2}\sin(kl)\left[S_i(2kl) - 2S_i(kl) \right]$$

$$+ \frac{1}{2}\cos(kl)\left[C + \ln\left(\frac{kl}{2}\right) + C_i(2kl) - 2C_i(kl) \right]$$

where C = Euler's Constant = 0.5772

$$C_i(x) = \text{Cosine Integral} = -\int_x^{\infty} \frac{\cos y}{y} dy = \int_{\infty}^{x} \frac{\cos y}{y} dy$$

$$S_i(x) = \int_0^{x} \frac{\sin y}{y} dy$$

$C_i(x)$ is related to $C_{in}(x)$

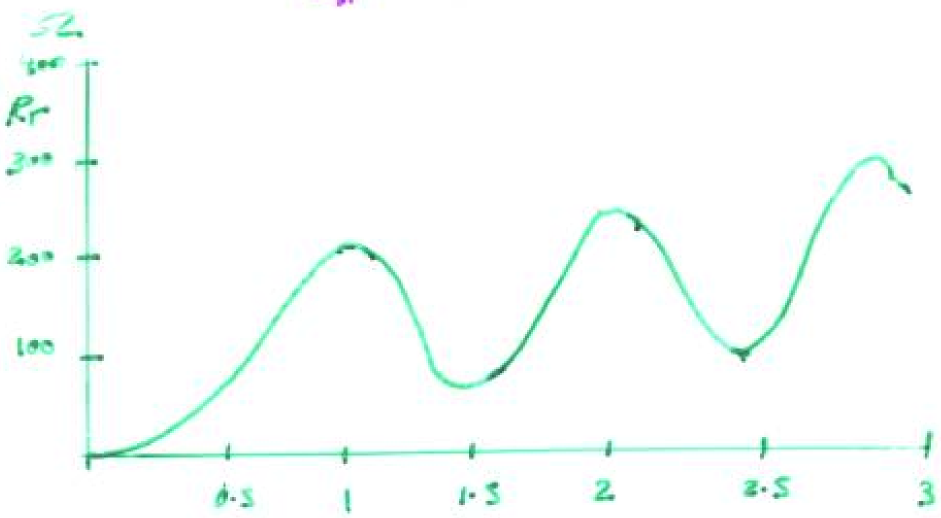
$$C_{in}(x) = \int_0^{x} \frac{1-\cos y}{y} dy = C + \ln(x) - C_i(x)$$

Radiation Resistance :

$$P_{rad} = \frac{1}{2} |I_0|^2 R_r$$

or $$R_r = \frac{2 P_{rad}}{|I_0|^2}$$

$$= \frac{\eta}{2\pi} Q \quad .$$



Directivity :

The general form of the directivity :

$$D_0 = \frac{F(\theta, \phi)\big|_{max}}{\frac{1}{4\pi} \int_0^{2\pi}\int_0^{\pi} F(\theta, \phi) \sin\theta \, d\theta \, d\phi}$$

Where $$U = U_0 F(\theta, \phi)$$

For a finite length dipole antenna,

$$U_0 = \eta \frac{|I_d|^2}{8\pi^2}$$

& $$F(\theta,\phi) = F(\theta) = \left[\frac{\cos(\frac{kl}{2}\cos\theta) - \cos(\frac{kl}{2})}{\sin\theta} \right]^2$$

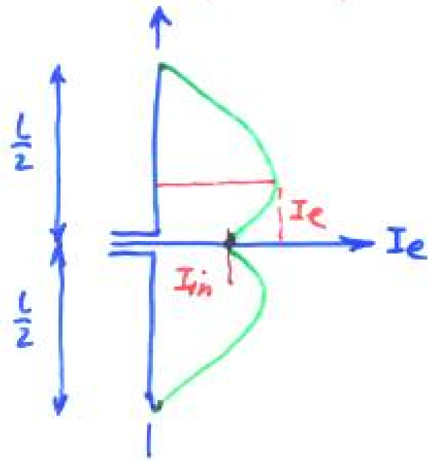
$$\therefore \quad D_0 = \frac{2\, F(\theta)|_{max}}{\int_0^{\pi} F(\theta) \sin\theta \, d\theta}$$

or $$D_0 = \frac{2\, F(\theta)|_{max}}{Q}.$$

& A_{em} = max. effective aperture

$$= \frac{\lambda^2}{4\pi} D_0$$

Finite length dipole input resistance:



$$P_{rad} = \frac{1}{2} |I_d|^2 R_r$$

$$= \frac{1}{2} |I_{in}|^2 R_{in}$$

$$I_{in} = I_0 \sin \frac{kl}{2}$$

$$\therefore \quad R_{in} = \frac{R_r}{\sin^2(\frac{kl}{2})}$$

Half-wavelength dipole:

$$kl = \frac{2\pi}{\lambda}\frac{\lambda}{2} = \pi \quad , \frac{kl}{2} = \frac{\pi}{2}$$

$$E_\theta \simeq j\eta \frac{I_0 e^{-jkr}}{2\pi r} \quad \frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}$$

$$H_\phi = E_\theta/\eta$$

$$W_{av} = \eta \frac{|I_0|^2}{8\pi^2 r^2}\left[\frac{\cos^2(\frac{\pi}{2}\cos\theta)}{\sin^2\theta}\right]$$

$$U = r^2 W_{av}$$

$$P_{rad} = \eta \frac{|I_0|^2}{4\pi}\int_0^\pi \frac{\cos^2(\frac{\pi}{2}\cos\theta)}{\sin\theta}\,d\theta$$

$$= \eta \frac{|I_0|^2}{8\pi}\int_0^{2\pi}\frac{1-\cos y}{y}\,dy = \eta \frac{|I_0|^2}{8\pi} C_{in}(2\pi)$$

$$C_{in}(2\pi) = 0.5772 + \ln(2\pi) - C_i(2\pi)$$

$$= 0.5772 + 1.838 - (-0.02) \simeq 2.435$$

$$D_0 = 4\pi \frac{U_{max}}{P_{rad}} = 4\pi \frac{U|_{\theta=\pi/2}}{P_{rad}}$$

$$= \frac{4}{C_{in}(2\pi)} = \frac{4}{2.435} \simeq 1.643$$

$$R_r = \frac{2P_{rad}}{|I_0|^2} = \frac{\eta}{4\pi} C_{in}(2\pi) = 30(2.435) = 73\ \Omega$$

$$\& \ A_{em} = \frac{\lambda^2}{4\pi} D_0 = \frac{\lambda^2}{4\pi}(1.643) = 0.13\lambda^2$$

Linear elements near infinite plane Conductors

Obstacles & their effects on the radiation properties.

The ground is the most significant obstacle. Any radiation directed towards the ground undergoes reflection which depends on:

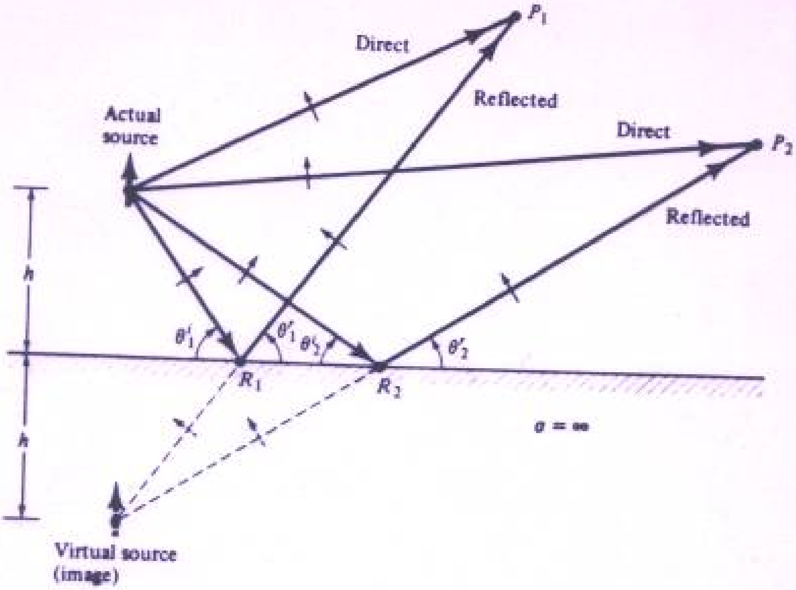
geometry & Constitutive parameters.

Methods of analysis include

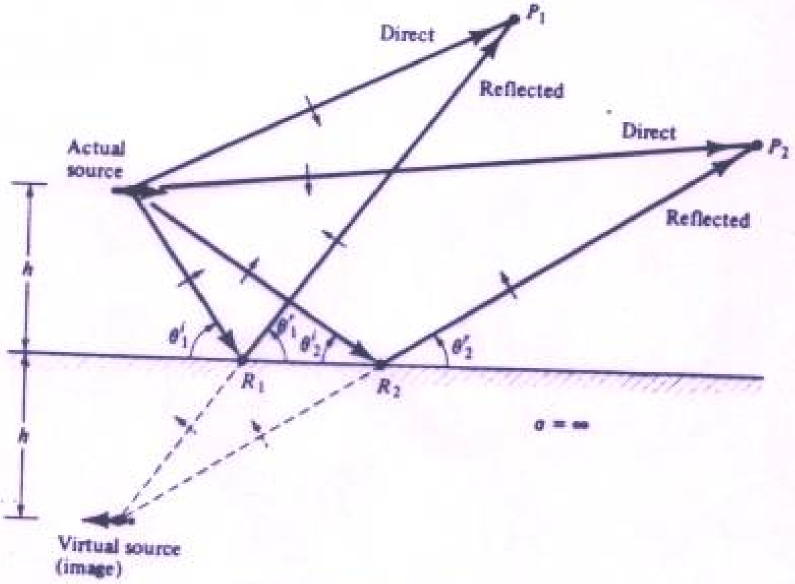
- Geometrical Theory of diffraction (GTD)
- Moments Method (MM)

Assuming an infinite ground Conductor, we use " Image theory ".

Virtual Sources, Combined with actual Sources form an equivalent systems

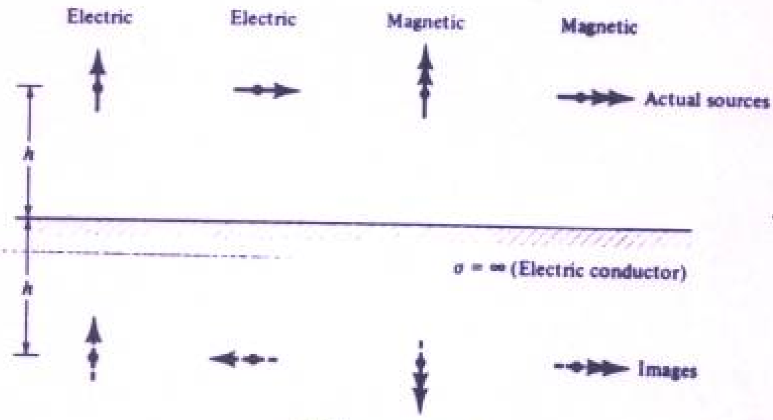


(a) Vertical electric dipole

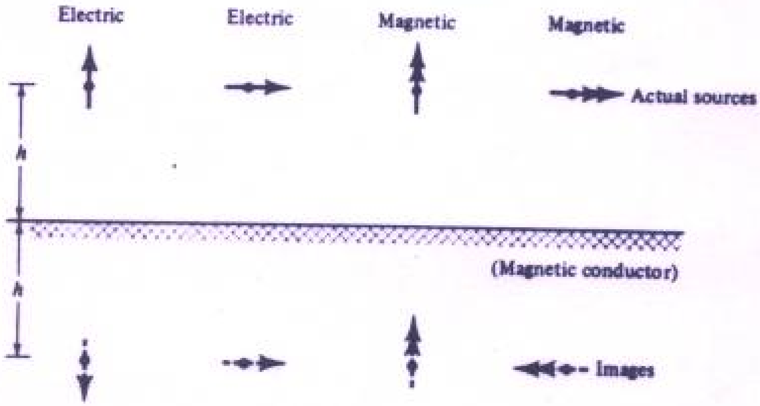


(b) Horizontal electric dipole

Figure 4.11 Vertical and horizontal electric dipoles above an infinite, flat, perfect electric conductor.



(a) Electric conductor



(b) Magnetic conductor

Figure 4.12 Electric and magnetic sources and their images near electric and magnetic conductors.