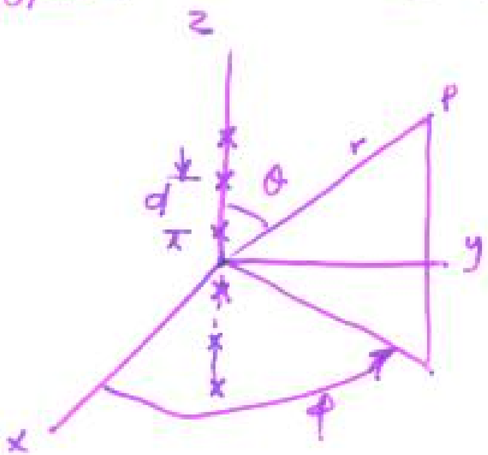
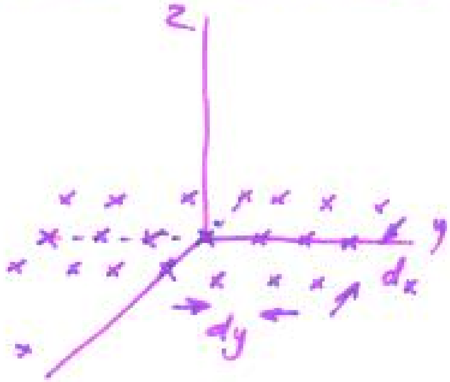
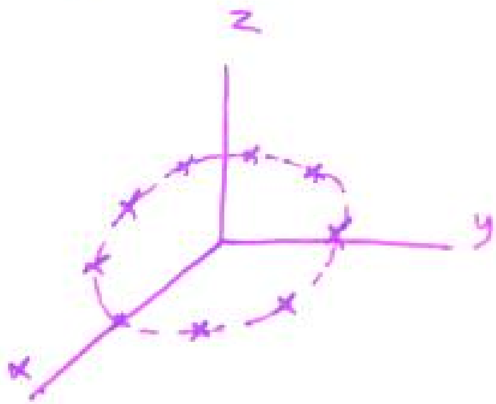
ANTENNA ARRAYS

Types! Linear , Planar , Circular





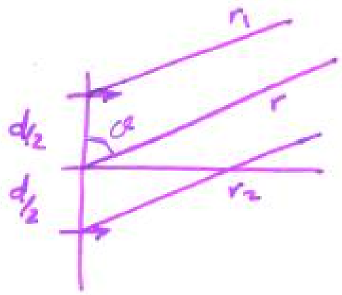


The Control variables in an array of identical elements!

1. Geometrical Configuration
2. Relative displacement between elements
3. Excitation amplitude
4. Excitation phase
5. Relative pattern of the individual elements.

Two - element Array

2 infinitesimal dipoles



$E_t = E_1 + E_2$

$$= \hat{a}_\theta \, j\eta \, \frac{kI_0 l}{4\pi} \left\{ \frac{e^{-j[kr_1 - \beta/2]}}{r_1} |\cos\theta_1| + \frac{e^{-j[kr_2 + \beta/2]}}{r_2} |\cos\theta_2| \right\}$$

$\beta \rightarrow$ the phase difference between the elements.

using the far field approximation

$\theta_1 \simeq \theta_2 \simeq \theta$

$\left.\begin{array}{l} r_1 \simeq r - \frac{d}{2}\cos\theta \\ r_2 \simeq r + \frac{d}{2}\cos\theta \end{array}\right\}$ phase

$r_1 \simeq r_2 \simeq r$ amplitude

$$\therefore E_t = \hat{a}_\theta \, j\eta \, \frac{kI_0 l e^{-jkr}}{4\pi r} |\cos\theta| \left[e^{j\left(\frac{kd\cos\theta + \beta}{2}\right)} + e^{-j\left(\frac{kd\cos\theta + \beta}{2}\right)} \right]$$

$$= \hat{a}_\theta \, j\eta \, \frac{kI_0 l e^{-jkr}}{4\pi r} |\cos\theta| \; 2\cos\left[\tfrac{1}{2}(kd\cos\theta + \beta)\right]$$

$=$ element factor $\times$ array factor

$$AF = 2\cos\left[\tfrac{1}{2}(kd\cos\theta + \beta)\right]$$

$$\text{or } AF_n = \cos\left[\tfrac{1}{2}(kd\cos\theta + \beta)\right]$$

Example: for the 2 element array, find the nulls of the total field when $d = \lambda/4$ and $\beta = 0, +\frac{\pi}{2}, -\frac{\pi}{2}$

a) $\beta = 0 \rightarrow E_{tn} = |\cos\theta| \cos\left(\frac{\pi}{4}\cos\theta\right)$

Nulls $\rightarrow E_{tn} = 0 = |\cos\theta| \cos\left(\frac{\pi}{4}\cos\theta\right)$

$\therefore \cos\theta_n = 0 \rightarrow \theta_n = 90^\circ$

also $\cos\left(\frac{\pi}{4}\cos\theta_n\right) = 0 \rightarrow \frac{\pi}{4}\cos\theta_n = \frac{\pi}{2}, -\frac{\pi}{2}$
does not exist.

$\therefore$ only one null at $\theta_n = 90^\circ$.

b) $\beta = \frac{\pi}{2} \rightarrow E_{tn} = |\cos\theta| \cos\left[\frac{\pi}{4}(\cos\theta + 1)\right]$

$\therefore \cos\theta_n = 0 \rightarrow \theta_n = 90^\circ$

also $\frac{\pi}{4}(\cos\theta_n + 1) = \frac{\pi}{2} \rightarrow \cos\theta_n = 1 \rightarrow \theta_n = 0^\circ$

c) $\beta = -\frac{\pi}{2} \longrightarrow E_{tn} = |\cos\theta| \cos\left[\frac{\pi}{4}(\cos\theta - 1)\right]$

$\therefore \cos\theta_n = 0 \rightarrow \theta_n = 90^\circ$

also $\frac{\pi}{4}(\cos\theta_n - 1) = -\frac{\pi}{2} \rightarrow \theta_n = 180^\circ$

General 2-element array nulls:

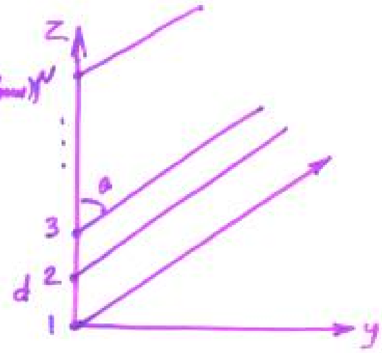
$E_{tn} = |\cos\theta| \cos\left[\frac{1}{2}(kd\cos\theta + \beta)\right]$

$\therefore E_{tn} = 0 \rightarrow \cos\theta_n = 0 \rightarrow \theta_n = 90^\circ$

Also when $\cos\left[\frac{1}{2}(kd\cos\theta + \beta)\right] = 0 \rightarrow \frac{1}{2}(kd\cos\theta + \beta) = \pm\frac{2n+1}{2}\pi$

$\therefore \theta_n = \cos^{-1}\left[\frac{1}{kd}\left\{-\beta \pm (2n+1)\pi\right\}\right] \quad n = 0, 1, 2, \ldots$

N-element Linear Array 'uniform'



$$AF = 1 + e^{j(kd\cos\theta+\beta)} + e^{j2(kd\cos\theta+\beta)} + \cdots\cdots e^{j(N-1)(kd\cos\theta+\beta)}$$

$$= \sum_{n=1}^{N} e^{j(n-1)(kd\cos\theta+\beta)}$$

or $$AF = \sum_{n=1}^{N} e^{j(n-1)\psi}, \quad \psi = kd\cos\theta+\beta.$$

$$= 1 + e^{j\psi} + e^{2j\psi} + \cdots e^{j(N-1)\psi}$$

$$AF\, e^{j\psi} = e^{j\psi} + e^{2j\psi} + e^{3j\psi} + \cdots e^{jN\psi}$$

$$\therefore AF = \frac{e^{jN\psi}-1}{e^{j\psi}-1} = e^{j\frac{N-1}{2}\psi}\,\frac{\sin\frac{N\psi}{2}}{\sin\frac{\psi}{2}}$$

Moving reference to the centre of array:

$$\therefore AF_n = \frac{1}{N}\,\frac{\sin\frac{N\psi}{2}}{\sin\frac{\psi}{2}}$$

& for small values of $\psi \rightarrow AF_n \simeq \dfrac{\sin\frac{N\psi}{2}}{\frac{N\psi}{2}}$

Nulls $\sin\frac{N\psi}{2} = 0 \rightarrow \frac{N\psi}{2} = \pm n\pi$

or $$\theta_n = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2n}{N}\pi\right)\right]$$

for $n = 1, 2, 3, \ldots$ & $n \neq N, 2N, \ldots$

Max. values occur $\rightarrow$ $\frac{\psi}{2} = \frac{1}{2}(kd\cos\theta + \beta) = \pm m\pi$

or $\theta_m = \cos^{-1}\left[\frac{\lambda}{2\pi d}(-\beta \pm 2m\pi)\right]$

$m = 0, 1, 2, \ldots$

for small ψ $\quad AF = \dfrac{\sin\frac{N\psi}{2}}{\frac{N\psi}{2}}$ $\rightarrow$ only one max.

at $\rightarrow$ $\theta_m = \cos^{-1}\left(\frac{\lambda\beta}{2\pi d}\right)$ (when $m = 0$)

3dB points $\quad \frac{N\psi}{2} = \frac{N}{2}(kd\cos\theta + \beta)\Big|_{\theta=\theta_h} = \pm 1.391$

$\therefore \theta_h = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2.782}{N}\right)\right]$

also can be written as $\quad \theta_h = \frac{\pi}{2} - \sin^{-1}\left[\frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2.782}{N}\right)\right]$

for large d $(d \gg \lambda)$ $\quad \theta_h \simeq \left[\frac{\pi}{2} - \frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2.782}{N}\right)\right]$

& HPBW $= 2|\theta_m - \theta_h|$

Maximum of 1st minor lobe $\rightarrow$ $\frac{N\psi}{2} \simeq \pm\frac{3\pi}{2}$

$\therefore \frac{N}{2}(kd\cos\theta + \beta) \simeq \pm\frac{3\pi}{2}$

or $\theta_s = \cos^{-1}\left\{\frac{\lambda}{2\pi d}\left[-\beta \pm \frac{3\pi}{N}\right]\right\}$

AF at this point $= \dfrac{\sin\frac{N\psi}{2}}{\frac{N\psi}{2}} = \frac{2}{3\pi} = 0.212$ $(= -13.46\,\text{dB})$

Broadside Arrays

AF max. occur when $\psi = 0 = kd\cos\theta + \beta$

for $\theta = 90° \rightarrow \beta = 0°$ for broadside condition

end fire arrays

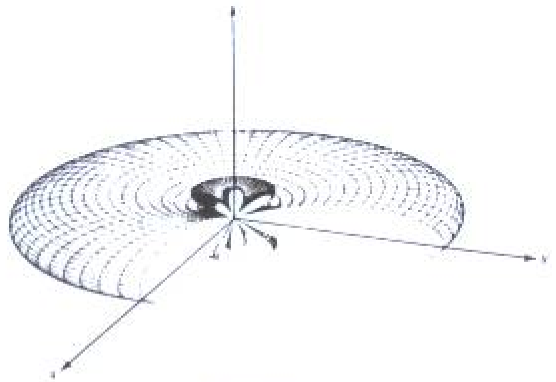
A F. maximum in the direction $\theta = 0°$ or $180°$.

$\therefore \psi = kd\cos\theta + \beta\Big|_{\theta = 0°} = kd + \beta \rightarrow \beta = -kd$

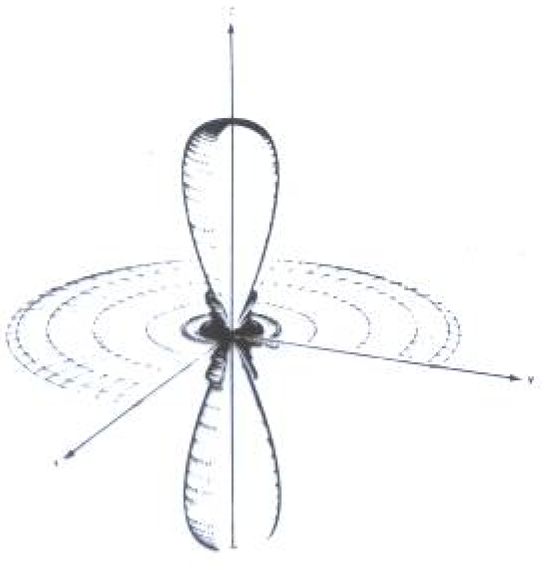
or $= kd\cos\theta + \beta\Big|_{180°} = -kd + \beta \rightarrow \beta = +kd$

phased (scanned) Arrays

$\psi = kd\cos\theta + \beta\Big|_{\theta = \theta_0} = 0 \rightarrow \quad \beta = -kd\cos\theta_0$



(a) Broadside



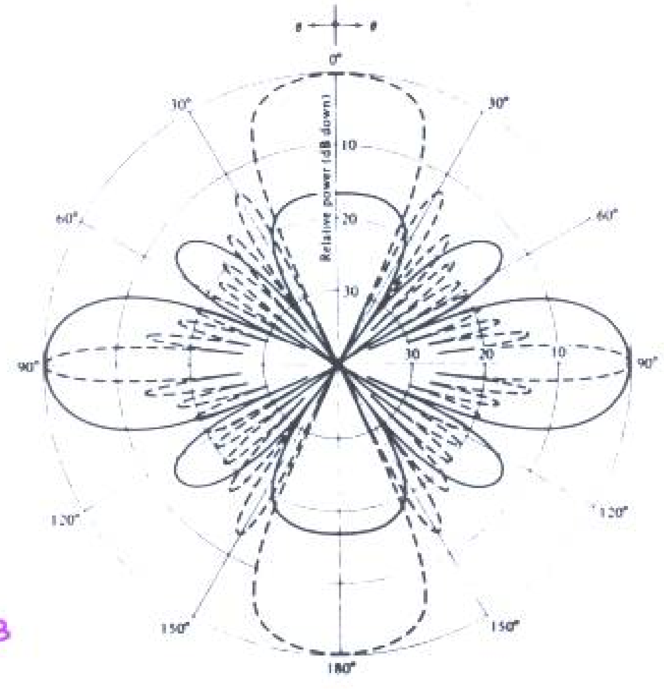
(b) Broadside/end-fire

Figure 6.6 Three-dimensional amplitude patterns for broadside, and broadside/end-fire arrays.

$$AF_n = \frac{\sin 5\psi}{10 \sin \frac{\psi}{2}}$$

$$\psi = kd\cos\theta + \beta$$

$$\beta = 0$$



———— $d = \lambda/4$

– – – – – $d = \lambda$

Figure 6.7 Array factor patterns of a 10-element uniform amplitude broadside array ($N = 10$, $\beta = 0$).

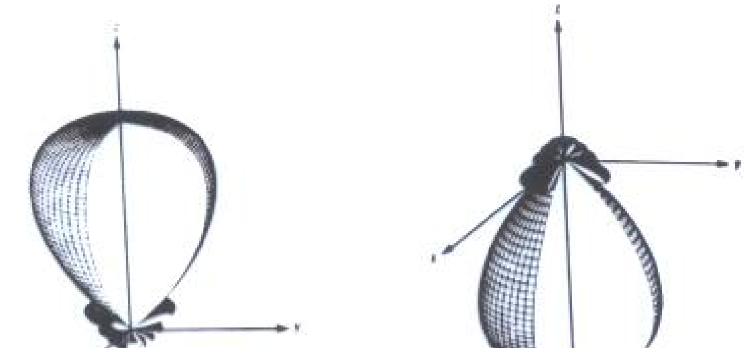
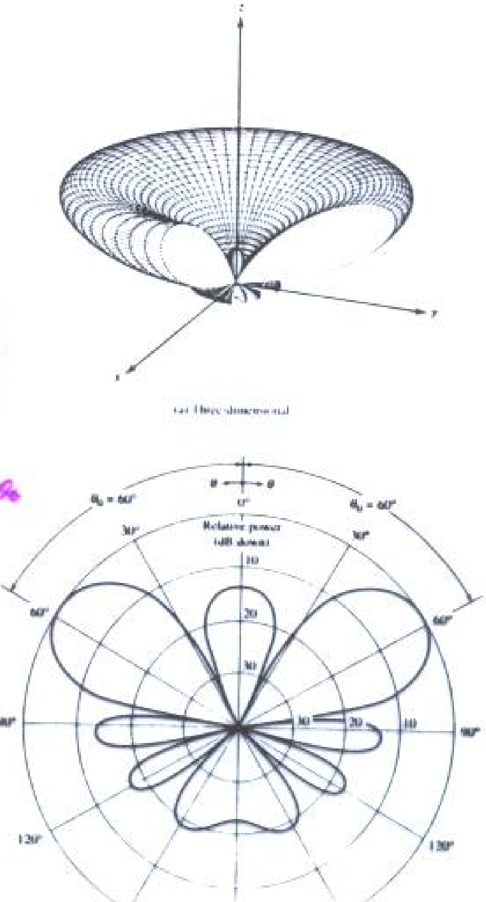


Figure 6.8 Three-dimensional amplitude patterns for end-fire arrays toward $\theta = 0°$ and 180°.

$$AF_n = \frac{\sin 5\psi}{10 \sin \frac{\psi}{2}}$$

$$\psi = kd\cos\theta + \beta$$
$$= kd\cos\theta - kd\cos\theta_0$$
$$d = \frac{\lambda}{4}$$
$$\psi = \frac{\pi}{2}\left(\cos\theta - \frac{1}{2}\right)$$



(b) Two-dimensional

Figure 6.10 Three- and two-dimensional array factor patterns of a 10-element uniform amplitude scanning array ($N = 10$, $\beta = -kd\cos\theta_0$, $\theta_0 = 60°$, $d = \lambda/4$.)

Directivity of B.S. & E.F. Linear Arrays

1. B.S. uniform array:

$$(AF)_n = \frac{\sin\left[\frac{N}{2}kd\cos\theta\right]}{N\sin\left[\frac{1}{2}kd\cos\theta\right]}$$

for $d \ll \lambda$ "Large array"

$$(AF)_n \simeq \frac{\sin\left[\frac{N}{2}kd\cos\theta\right]}{\left(\frac{N}{2}kd\cos\theta\right)}$$

Radiation intensity $\rightarrow U(\theta) = \left[(AF)_n\right]^2 = \left[\frac{\sin Z}{Z}\right]^2$

where $Z = \frac{N}{2}kd\cos\theta$

$\therefore U_{max} = 1$ for $\theta_{max} = \pi/2$

& $U_0 = \frac{1}{4\pi}P_{rad} = \frac{1}{2}\int_0^{\pi}\left[\frac{\sin Z}{Z}\right]^2 \sin\theta\, d\theta$

$$= \frac{1}{2}\int_0^{\pi}\left[\frac{\sin\left(\frac{N}{2}kd\cos\theta\right)}{\left(\frac{N}{2}kd\cos\theta\right)}\right]^2 \sin\theta\, d\theta$$

change variable to $Z = \frac{N}{2}kd\cos\theta$

$$\therefore dZ = -\frac{N}{2}kd\sin\theta\, d\theta$$

$$\therefore U_0 = -\frac{1}{Nkd}\int_{\frac{Nkd}{2}}^{-\frac{Nkd}{2}}\left[\frac{\sin Z}{Z}\right]^2 dZ = \frac{1}{Nkd}\int_{-\frac{Nkd}{2}}^{\frac{Nkd}{2}}\left[\frac{\sin Z}{Z}\right]^2 dZ$$

For a large array i.e. $\frac{Nkd}{2} \rightarrow$ large

$$\therefore \quad U_0 \simeq \frac{1}{Nkd} \int_{-\infty}^{\infty} \left[\frac{\sin Z}{Z} \right]^2 dz$$

using $\int_{-\infty}^{\infty} \left[\frac{\sin Z}{Z} \right]^2 dz = \pi$

$$\therefore \quad U_0 \simeq \frac{\pi}{Nkd} \longrightarrow D_0 = \frac{U_{max}}{U_0} = \frac{Nkd}{\pi}$$

$$\text{or } D_0 = 2N\frac{d}{\lambda}$$

For a linear array with equal element spacing

$$L = (N-1)d$$

$$\therefore D_0 = 2N\frac{d}{\lambda} = 2\left(1+\frac{L}{d}\right)\frac{d}{\lambda}$$

if $L \gg d \quad \therefore D_0 \simeq 2\frac{L}{\lambda}$

2. End-fire uniform array:

$$\psi = kd(\cos\theta - 1)$$

$$(AF)_n = \frac{\sin\left[\frac{N}{2}kd(\cos\theta-1)\right]}{\left[\frac{N}{2}kd(\cos\theta-1)\right]}$$

$$\therefore U(\theta) = [(AF)_n]^2 = \left[\frac{\sin Z}{Z}\right]^2$$

where $Z = \frac{N}{2}kd(\cos\theta - 1)$

$$U_0 = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi}\left[\frac{\sin Z}{Z}\right]^2 \sin\theta\, d\theta\, d\phi$$

$$= \frac{1}{2}\int_0^{\pi}\left[\frac{\sin\left(\frac{N}{2}kd(\cos\theta-1)\right)}{\frac{N}{2}kd(\cos\theta-1)}\right]^2 \sin\theta\, d\theta$$

$$Z = \frac{N}{2}kd(\cos\theta - 1)$$

$$dZ = -\frac{N}{2}kd\sin\theta\, d\theta$$

$$\therefore U_0 = -\frac{1}{Nkd}\int_0^{-Nkd}\left[\frac{\sin Z}{Z}\right]^2 dZ = \frac{1}{Nkd}\int_0^{Nkd}\left[\frac{\sin Z}{Z}\right]^2 dZ$$

using the condition of large array:

$$\therefore U_0 \simeq \frac{1}{Nkd}\int_0^{\infty}\left[\frac{\sin Z}{Z}\right]^2 dZ = \frac{\pi}{2Nkd}$$

& $D_0 = \frac{U_{max}}{U_0} = \frac{2Nkd}{\pi} = 4N\frac{d}{\lambda}$

using $L = (N-1)d$

$$\therefore D_0 = 4\left(1+\frac{L}{d}\right)\frac{d}{\lambda}$$

& for $L \gg d$ $\qquad D_0 \simeq 4\frac{L}{\lambda}$

$\therefore$ D_0 for E.F. array = Twice D_0 for B.S. array.