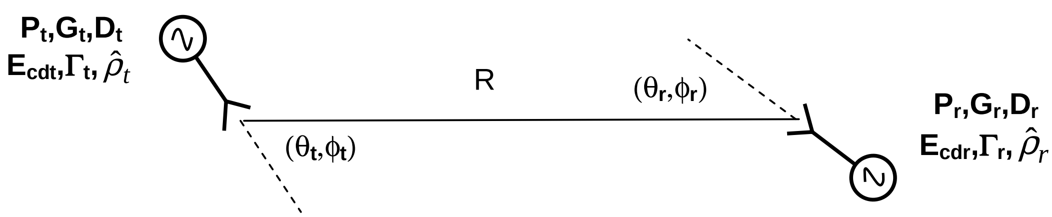
ANTENNA PARAMETERS

Friis Transmission Equation

Establishes the relationship between the received power in a receiving antenna and the transmitted power from a transmitting antenna, separated by a distance R.



$$W_o = \frac{P_t}{4\pi R^2} e_t$$

$$W_t = \frac{P_t}{4\pi R^2} e_t D_t(\theta_t, \phi_t)$$

$$A_r = e_r D_r(\theta_r, \phi_r)\left(\frac{\lambda^2}{4\pi}\right)$$

$$P_r = W_t A_r = P_t e_t e_r \frac{\lambda^2 D_t D_r}{(4\pi R)^2} |\hat{\rho}_t \cdot \hat{\rho}_r|^2$$

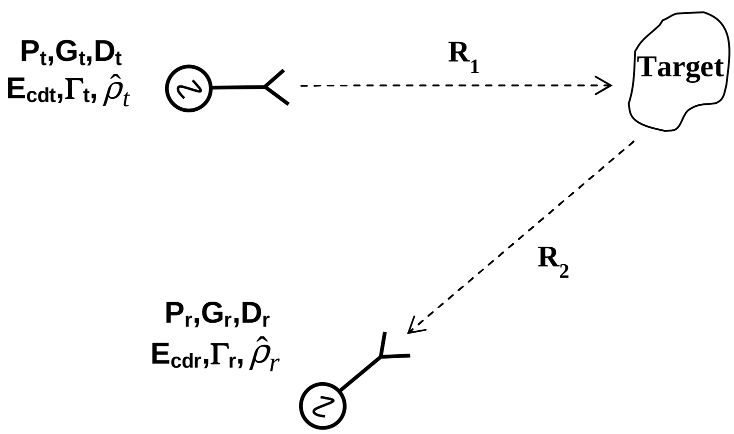
$$\frac{P_r}{P_t} = e_t e_r D_t D_r \left(\frac{\lambda}{4\pi R}\right)^2 |\hat{\rho}_t \cdot \hat{\rho}_r|^2$$

For reflection and polarization matched antennas, aligned for maximum transmission and reception

$$\therefore \frac{P_r}{P_t} = \left(\frac{\lambda}{4\pi R}\right)^2 G_{ot}\, G_{or}$$

Where $\left(\frac{\lambda}{4\pi R}\right)^2$ → **Free space loss factor.**

Radar Range Equation



A target radar cross section (also known as echo area) σ may be define as" ***The area intercepting that amount of power which when scattered isotropically, produces as the receiver a density which is equal to that scattered by the actual target*"****.**

In equation form → $$\lim_{R\to\infty}\left[\frac{\sigma W_i}{4\pi R^2}\right] = W_s$$

Or → $$\sigma = \lim_{R\to\infty}\left[4\pi R^2 \frac{W_s}{W_i}\right] = W_s$$

σ is the radar cross section of the target in m^2

R is the observation distance from target

Wi and Ws are the incident and scattered power densities.

The captured power by a target may be expressed as:

$$P_c = \sigma W_t = \sigma \frac{P_t G_t(\theta_t, \phi_t)}{4\pi R_1^{\ 2}} = e_t \sigma \frac{P_t D_t(\theta_t, \phi_t)}{4\pi R_1^{\ 2}}$$

The power captured by the target is reradiated isotropically. The scattered power density is then given by:

$$W_s = \frac{P_c}{4\pi R_2^{\ 2}} = e_t \sigma \frac{P_c D_t(\theta_t, \phi_t)}{(4\pi R_1 R_2)^2}$$

The power delivered to the receiver is given by:

$$P_r = A_r W_s = e_t e_r \sigma \frac{P_t D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r)}{4\pi} \left(\frac{\lambda}{4\pi R_1 R_2} \right)^2$$

The ratio of power received to the power transmitted will be given by:

$$\frac{P_r}{P_t} = e_t e_r \sigma \frac{D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r)}{4\pi} \left(\frac{\lambda}{4\pi R_1 R_2} \right)^2 .PLF$$

For polarization matched antennas aligned for maximum directional radiation and reception, the previous equation reduces to :

$$\frac{P_r}{P_t} = \sigma \frac{G_{ot} G_{or}}{4\pi} \left(\frac{\lambda}{4\pi R_1 R_2} \right)^2$$

This is the radar range equation.

Antenna Noise Temperature

Every object with a physical temperature above absolute zero (0 K = -273 °C) radiates energy. This is represented by an equivalent temperature T_B, known as brightness temperature

$$T_B(\theta,\phi)=\varepsilon(\theta,\phi)T_m=(1-|\Gamma|^2)T_m$$

T_B → Brightness temperature
ε → emmissivity
T_m → molecular temperature
$\Gamma(\theta,\phi)$ → reflection coefficient of the surface.

The brightness temperature emitted by the different sources is intercepted by the antenna and appears at the terminals of the antenna as antenna temperature T_A:

$$T_A=\frac{\int_0^{2\pi}\int_0^{\pi}T_B(\theta,\phi)G(\theta,\phi)\sin\theta\,d\theta\,d\phi}{\int_0^{2\pi}\int_0^{\pi}G(\theta,\phi)\sin\theta\,d\theta\,d\phi}$$

where $G(\theta,\phi)$ is the gain pattern of the antenna.

The combined effect of the antenna temperature and the transmission line temperature can be found from:

$$T_a=T_A e^{-2\alpha l}+T_{AP}e^{-2\alpha l}+T_o\left(1-e^{-2\alpha l}\right)$$

where:

T_a is the antenna temperature at the receiver terminals

T_{AP} Antenna temperature at its terminals due to the physical temperature

T_P The antenna physical temperature

α is the attenuation coefficient

l is the length of the transmission line or waveguide

$T_{AP} = \left(\frac{1}{e_A} - 1 \right) T_P$, where e_A is the thermal efficiency of the antenna.