ANTENNA PARAMETERS

Antenna Input Impedance

The maximum power delivered to the antenna occurs when we have conjugate matching,

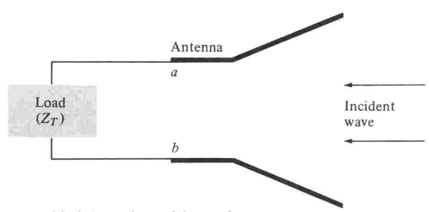
$$R_r + R_L = R_g \qquad and \qquad X_A = -X_g$$

Under maximum power transfer condition, we get:

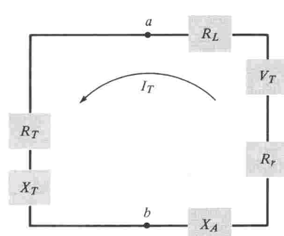
$$P_r = \frac{|V_g|^2}{2}\left[\frac{R_r}{4(R_r + R_L)^2}\right] = \frac{|V_g|^2}{8}\left[\frac{R_r}{(R_r + R_L)^2}\right]$$

Receiving Antenna Equivalent Circuit

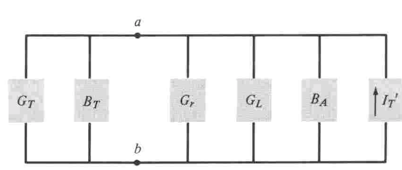
The receiving antenna equivalent circuit is shown in the following Figure.



(a) Antenna in receiving mode



(b) Thevenin equivalent



(c) Norton equivalent

The power delivered to the load impedance is given by:

$$P_T = \frac{|V_T|^2}{8}\left[\frac{R_T}{(R_r + R_L)^2}\right] = \frac{|V_T|^2}{8}\left[\frac{1}{(R_r + R_L)}\right] = \frac{|V_T|^2}{8R_T}$$

Antenna Equivalent Areas

There are different equivalent areas (or apertures) that can be defined for an antenna, when it is used in the receiving mode. The most important among them is the effective area (effective aperture). The other equivalent areas are the scattering area, the loss area and the capture area. Also, we can define a physical area or physical aperture, which is most important for aperture antennas.

Effective area

Is defined as the ratio of the available power at the terminals of the receiving antenna to the power flux density of a plane wave incident on the antenna from a given direction.

$$A_e = \frac{P_T}{W_i} = \frac{(1/2)|I_T|^2 R_T}{W_i}$$

Where A_e is the effective area (aperture) m^2

P_T is the power delivered to the load W,

Wi is the power density of the incident wave (W/m^2).

The effective aperture can be expressed in terms of the antenna impedance,

$$A_e = \frac{|V_T|^2}{2W_i}\left[\frac{R_T}{(R_r + R_L + R_T)^2 + (X_A + X_T)^2}\right]$$

Under the condition of maximum power transfer, the effective aperture becomes the maximum effective aperture,

$$A_{em} = \frac{|V_T|^2}{8W_i}\left[\frac{R_T}{(R_r + R_L)^2}\right] = \frac{|V_T|^2}{8W_i}\left[\frac{1}{(R_r + R_L)}\right]$$

The scattering, loss, and capture areas are given by:

$$A_s = \frac{|V_T|^2}{8W_i}\left[\frac{R_r}{(R_r + R_L)^2}\right]$$

$$A_L = \frac{|V_T|^2}{8W_i}\left[\frac{R_L}{(R_r + R_L)^2}\right]$$

and $$A_c = \frac{|V_T|^2}{8W_i}\left[\frac{R_T + R_r + R_L}{(R_r + R_L)^2}\right]$$

A very important parameter is the aperture efficiency, which is defined as the ratio of maximum effective aperture to thew physical aperture

$$\varepsilon_{ap} = \frac{A_{em}}{A_p}$$

Example

A uniform plane wave is incident upon a very short dipole ($l \ll \lambda$). Find the maximum effective aperture, assuming that the radiation resistance of the very short dipole is $R_r = 80(\pi l/\lambda)^2$, and that the incident field is linearly polarized along the axis of the dipole.

For $R_L = 0$, then the maximum effective aperture is given by:

$$A_{em} = \frac{|V_T|^2}{8W_i}\left[\frac{1}{R_r}\right]$$

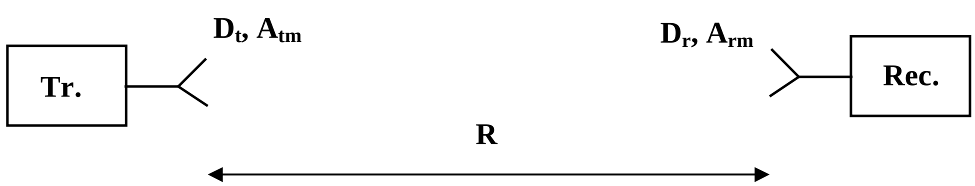
The induced voltage on the dipole is VT=El

$$W_i = \frac{E^2}{2\eta_o}$$

Where η_o is the intrinsic impedance of free space = 120 π Ω.

Thus $$A_{em} = \frac{(El)^2}{8(E^2/2\eta_o)(80\pi^2 l^2/\lambda^2)} = \frac{3\lambda^2}{8\pi} = 0.119\lambda^2$$

Relationship of Maximum Directivity and Maximum Effective Aperture



$W_o = \dfrac{P_t}{4\pi R^2}$ Where W_o is the power density at the receiving antenna, assuming the transmitting antenna is a point source.

$W_t = W_o\, D_t = \dfrac{P_t D_t}{4\pi R^2}$ Where W_t is the power density at the receiving antenna, due to the actual transmitting antenna.

The received power is then given by:

$$P_r = W_t\, A_r = \frac{P_t D_t A_r}{4\pi R^2} \quad \rightarrow \quad D_t A_r = \frac{P_r}{P_t}(4\pi\, R^2)$$

If we exchange antennas 1 and 2 (due to Reciprocity)

$$\therefore also\ we\ get \quad \rightarrow \quad D_r A_t = \frac{P_r}{P_t}(4\pi\, R^2)$$

$$\therefore\ D_r A_t = D_t A_r \quad \rightarrow \quad \frac{D_t}{A_t} = \frac{D_r}{A_r}$$

Also $$\frac{D_{ot}}{A_{tm}} = \frac{D_{or}}{A_{rm}}$$

Where "o" denotes the maximum directivity and "m" the maximum effective aperture.

To find the constant of proportionality, we consider a typical antenna element such as the short dipole. Its maximum effective aperture can be found = $\frac{3\lambda^2}{8\pi}$**, and its maximum directivity is** $\frac{3}{2}$**.**

$$\therefore\ \frac{D_o}{A_{em}} = \frac{4\pi}{\lambda^2} \quad \rightarrow \quad D_o = \frac{4\pi}{\lambda^2} A_{em}$$

If we take the antenna efficiency into consideration, then the maximum directivity of the antenna is replaced by the maximum power gain.

$$G_o = \frac{4\pi}{\lambda^2} A_{em}$$