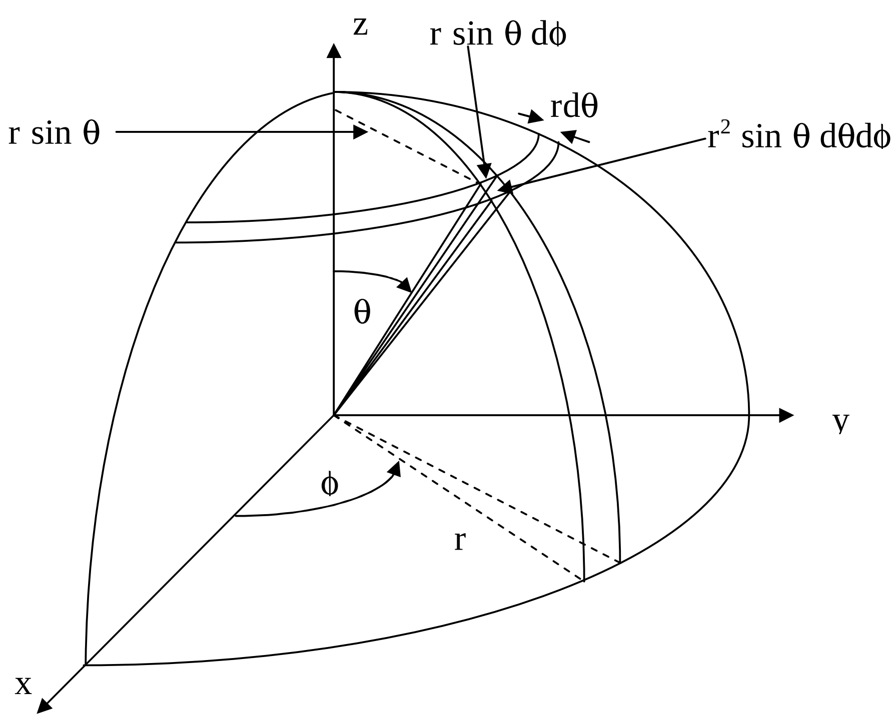
FUNDAMENTAL PARAMETERS OF ANTENNAS

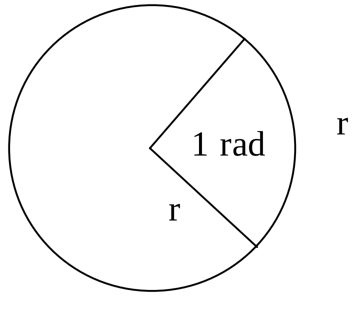
- Radiation pattern
- Power density & radiation intensity
- Directivity
- Directional, Omni-directional, and isotropic patterns
- Gain
- Efficiency
- Half-power beam-width (HPBW)
- Band-with
- Polarisation and polarisation loss factor (PLF)
- Impedance
- Effective aperture

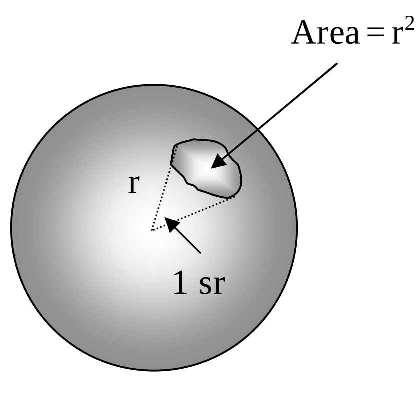
- Transmission equation
- Radar range equation & radar cross section (RCS)
- Antenna temperature

SPHERICAL COORDINATE SYSTEM



RADIAN AND STERADIAN





$$dA = r^2 \sin\theta \, d\theta \, d\phi \quad m^2$$

$$d\Omega = \frac{dA}{r^2} = \sin\theta \, d\theta \, d\phi \quad sr$$

The instantaneous Poynting vector is defined as:

$\mathrm{W} = \mathrm{E} \, X \, \mathrm{H}$ Where E is the instantaneous E field in V/m and H is the instantaneous H field in A/m.

The total radiated power can be obtained from:

$$\mathrm{P} = \oiint \mathrm{W} \cdot ds = \oiint \mathrm{W} \cdot \hat{n} \, da$$

When time dependence is harmonic, i.e. $e^{i\omega t}$

Then $W_{av} = \frac{1}{2} \mathrm{Re}[E \; X \; H^*]$

And the radiated power becomes:

$$P_{rad} = \oiint W_{av} \cdot ds = \frac{1}{2} \oiint \mathrm{Re}[EXH^*] \cdot ds$$

For a point source radiator, the radiated power can be calculated as follows:

$$P_{rad} = \oiint W_0 \cdot ds = \int_0^{2\pi} \int_0^{\pi} W_0 \hat{a}_r \cdot \hat{a}_r \, r^2 \sin\theta \; d\theta \; d\phi$$

$$P_{rad} = \oiint W_0 \cdot ds = r^2 W_0 \int_0^{2\pi} \int_0^{\pi} \sin\theta \; d\theta \; d\phi = 2\pi r^2 W_0 \int_0^{\pi} \sin\theta \, d\theta = 4\pi r^2 W_0$$

$W_0 = \frac{P_{rad}}{4\pi r^2} \; W/m^2$ Uniformly distributed over the surface of the sphere.

Radiation intensity → Power radiated per unit solid angle

$U = r^2 W_{rad}$ where $U \rightarrow Rad.intensity\ in W/sr$

and $W_{rad} \rightarrow Rad.\ power\ density\ in W/m^2$

Example: If the radiation power density is given by:

$$\vec{W}_{rad} = \hat{a}_r W_r = \hat{a}_r A_o \frac{\sin\theta}{r^2} \ W/m^2$$

Find the total radiated power.

Solution: $U = A_o \frac{\sin\theta}{r^2} r^2 = A_o \sin\theta \ W/sr$

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} A_o \sin\theta \sin\theta \ d\theta d\phi$$

$$= 2\pi A_o \int_0^{\pi} \sin^2\theta \, d\theta = 2\pi A_o \frac{\pi}{2} = \pi^2 A_o \ W$$

MAXIMUM DIRECTIVITY

Directivity → Ratio of the radiation Intensity in a given direction to the average radiation intensity (i.e. U of an isotropic radiator radiating the same radiated power.)

$$D_g(\theta,\phi) = \frac{U(\theta,\phi)}{U_o} = \frac{4\pi U(\theta,\phi)}{P_{rad}}$$

Max. Directivity → The maximum value of directivity function.

$$Max.Directivity = D_o = \frac{U_{max}}{U_o} = \frac{4\pi U_{max}}{P_{rad}}$$

Example: Find the directivity of the antenna given in the previous example with $U = A_o \sin\theta \, W/sr$

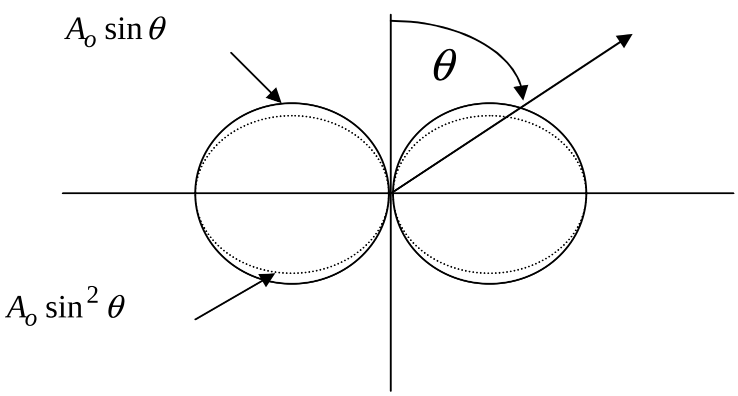
Solution: $U_{\max} = A_o$, $P_{rad} = \pi^2 A_o$

$$\therefore D_o = \frac{4\pi A_o}{\pi^2 A_o} = \frac{4}{\pi} = 1.273$$

Example: Compare the previous directivity to that of an antenna having $U = A_o \sin^2\theta \, W/sr$

Solution: $U_{\max} = A_o$, $P_{rad} = \int_0^{2\pi}\int_0^{\pi} A_o \sin^2\theta \sin\theta \; d\theta d\phi = 2\pi\, A_o(\frac{4}{3}) = \frac{8\pi}{3} A_o$

$$\therefore D_o = \frac{4\pi A_o}{\frac{8\pi}{3} A_o} = 1.5$$



GAIN:

The power gain of an antenna is defined similar to the directive gain except that the reference antenna , which is the isotropic radiator, has the same amount of input power.

$$G(\theta,\phi) = \frac{U(\theta,\phi)}{U_{ref}} = \frac{4\pi U(\theta,\phi)}{P_{in}} = \frac{P_{rad}}{P_{in}} \frac{4\pi U(\theta,\phi)}{P_{rad}} = \eta\, D_g(\theta,\phi)$$

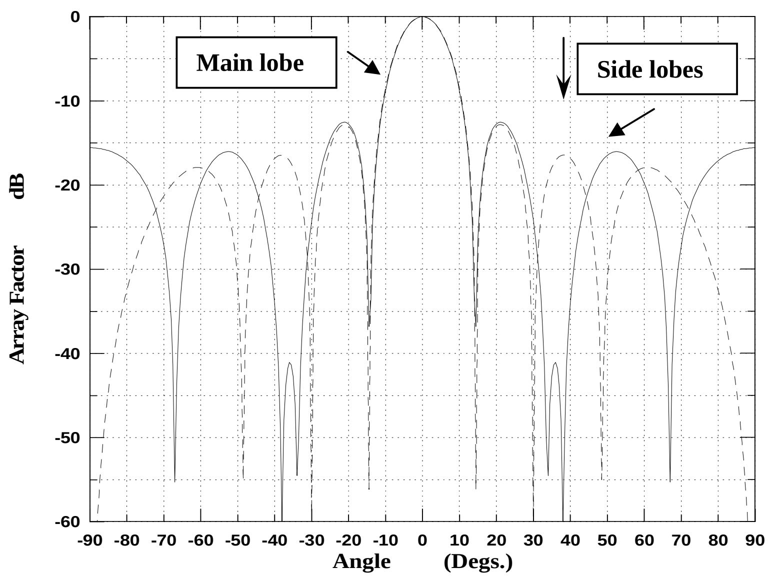
Usually the gain refers to the maximum power gain;

i.e. $G_o = \eta D_o$

where η is the antenna efficiency.

RADIATION PATTERN

A plot (normally a three dimensional one) of the radiation intensity, power density, or radiated field as a function of direction in space.



A more general expression of directivity:

$$U = B_0 F(\theta,\phi) \cong \frac{1}{2\eta}\left[\left|E_\theta(\theta,\phi)\right|^2 + \left|E_\phi(\theta,\phi)\right|^2 \right]$$

where B_0 is a constant, and E_θ *and* $E\phi$ are the antenna's far zone electric field components.

$$U_{\max} = B_0 F(\theta,\phi)\Big|_{\max} = B_0 F_{\max}(\theta,\phi)$$

$$P_{rad} = \oiint_\Omega U(\theta,\phi)\, d\Omega = B_0 \int_0^{2\pi}\int_0^{\pi} F(\theta,\phi) \sin\theta \, d\theta \, d\phi$$

The general expression of directivity can then be written as:

$$D(\theta,\phi)=4\pi\frac{F(\theta,\phi)}{\int_0^{2\pi}\int_0^{\pi}F(\theta,\phi)\sin\theta\,d\theta\,d\phi}$$

and the maximum directivity becomes:

$$D_0=4\pi\frac{F(\theta,\phi)\big|_{max}}{\int_0^{2\pi}\int_0^{\pi}F(\theta,\phi)\sin\theta\,d\theta\,d\phi}$$

$$D_0=\frac{4\pi}{\left[\int_0^{2\pi}\int_0^{\pi}F(\theta,\phi)\sin\theta\,d\theta\,d\phi\right]/F(\theta,\phi)\big|_{max}}=\frac{4\pi}{\Omega_A}$$

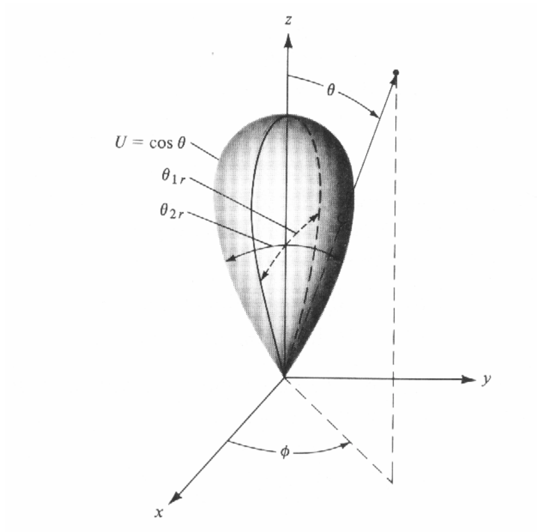
Where Ω_A is the beam solid angle and is given by:

$$\Omega_A=\frac{1}{F(\theta,\phi)\big|_{max}}\int_0^{2\pi}\int_0^{\pi}F(\theta,\phi)\sin\theta\,d\theta\,d\phi=\int_0^{2\pi}\int_0^{\pi}F_n(\theta,\phi)\sin\theta\,d\theta\,d\phi$$

Ω_A is defined as the solid angle through which all the antenna power will flow if $U(\theta,\phi)$ is constant and equal to its maximum value.

Simple expression for the maximum directivity can be used:

$$D_0=\frac{4\pi}{\Omega_A}=\frac{4\pi}{\theta_{1r}\theta_{2r}}=\frac{4\pi(180/\pi)^2}{\theta_{1d}\theta_{2d}}=\frac{41253}{\theta_{1d}\theta_{2d}}$$



For planar arrays, better approximation is

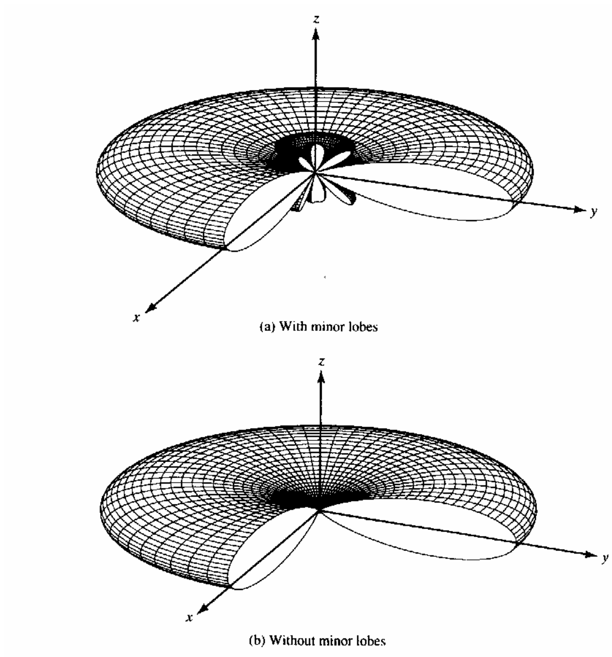
$$D_0 = \frac{32400}{\theta_{1d}\theta_{2d}}.$$

Omni-directional Patterns:

These patterns may be described by:

$$U(\theta,\phi) = \left|\sin^n(\theta)\right| \qquad 0 \le \theta \le \pi \text{ and } 0 \le \phi \le 2\pi$$

where n is an integer or non-integer value.



Approximate directivity for omni-directional patterns may be obtained:

$$D_0 \simeq \frac{101}{HPBW(\deg.) - 0.0027\left[HPBW\right]^2}$$

suitable for patterns with minor lobes (e.g. array factor of broadside array).

Another approximate expression, suitable for patterns with negligible minor lobes, was also derived:

$$D_0 \simeq -172.4 + 191\sqrt{0.818 + \frac{1}{HPBW(\deg.)}}\ .$$

Example:

Design an antenna with omni-directional amplitude pattern with HPBW of 90°. Express the radiation intensity by $U = \sin^n \theta$. Determine the value of n and attempt to identify elements that exhibit such a pattern. Determine the directivity of the antenna using the exact formula and the two approximate expressions.

Solution:

HPBW = 90o, angle at which half-power is $\theta = 45^0$

$$\therefore U(\theta = 45^0) = 0.5 = \sin^n(45^0) = 0.707^n$$

n = 2.

The element that has this type of pattern is an infinitesimal dipole. (As shown in chapter 4)

$$U_{max} = 1,\ P_{rad} = \int_0^{2\pi}\int_0^{\pi} \sin^2\theta \sin\theta \, d\theta \, d\phi = \frac{8\pi}{3}$$

$$D_0 = \frac{4\pi}{8\pi/3} = 1.5\ (= 1.761 dB)$$

Using the approximate formulae:

1) D_0 = 101/[90 – 0.0027(90)^2] = 1.4825   (= 1.71 dB)

2) D_0 = -172.4 + 191√0.818 + (1/90) = 1.516 (= 1.807 dB).