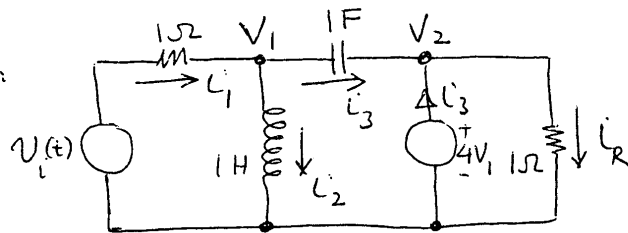


Problem 1:



Write the differential equation for each energy storage element.

$$\frac{di_2}{dt} = v_2 \quad ; \quad \frac{dv_3}{dt} = i_3$$

Therefore, the state vector is, $x = \begin{bmatrix} i_2 \\ v_3 \end{bmatrix}$

Now obtain v_1 , and i_3 in terms of the state variables,

$$\begin{aligned} v_1 &= v_3 + v_4 = v_3 + i_R = v_3 + i_3 + 4v_1 = v_3 + (i_2 - i_2) + 4v_1 \\ &= v_3 + i_2 - i_2 + 4v_1 \Rightarrow \end{aligned}$$

$$\therefore v_1 = \frac{1}{2} i_2 - \frac{1}{2} v_3 - \frac{1}{2} v_1$$

$$i_3 = i_1 - i_2 = v_1 - v_1 - i_2 = v_1 - \frac{1}{2} i_2 + \frac{1}{2} v_3 + \frac{1}{2} v_1 - i_2$$

$$\therefore i_3 = -\frac{3}{2} i_2 + \frac{1}{2} v_3 + \frac{3}{2} v_1$$

$$\text{Also, the output is } i_R = i_3 + 4v_1 = \frac{1}{2} i_2 - \frac{3}{2} v_3 - \frac{1}{2} v_1$$

Therefore,

$$\dot{x} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{3}{2} & \frac{1}{2} \end{bmatrix} x + \begin{bmatrix} -\frac{1}{2} \\ \frac{3}{2} \end{bmatrix} v_1 \quad ; \quad y = \begin{bmatrix} \frac{1}{2} & -\frac{3}{2} \end{bmatrix} x - \frac{1}{2} v_1$$

Problem 2:

The input potentiometer slider position r is converted to a voltage with a potentiometer identical to the output potentiometer.

$$V_r = K_p r$$

$$V_1 = A_1 (V_r - V_0) = A_1 K_p (r - \theta)$$

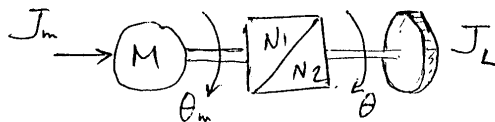
$$\textcircled{1} \quad V_2 = A_2 V_1 = A_1 A_2 K_p (r - \theta) \quad \text{motor voltage}$$

$$\theta = \frac{N_1}{N_2} \theta_m$$

θ_m is the motor shaft angle
 θ is angular position of the antenna
 $N_1 \ll N_2$

Applying Laplace transform yields:

$$V_2(s) = A_1 A_2 K_p (R(s) - \Theta(s))$$



$$J_e = J_m + \left(\frac{N_1}{N_2}\right)^2 J_L$$

$$K_T i_A = J_e \frac{d^2 \theta_m}{dt^2}$$

$$\theta = \frac{N_1}{N_2} \theta_m$$

$$V_2 = R_A i_A + K_v \frac{d\theta_m}{dt}$$

$$V_2 = \frac{R_A}{K_T} J_e \frac{d^2 \theta_m}{dt^2} + K_v \frac{d\theta_m}{dt}$$

$$V_2 = \frac{N_2}{N_1} \left[\frac{R_A J_e}{K_t} \frac{d^2 \theta}{dt^2} + K_v \frac{d\theta}{dt} \right]$$

$$V_2 = \frac{N_2}{N_1} K_v \left[\frac{R_A J_e}{K_v K_t} \frac{d^2 \theta}{dt^2} + \frac{d\theta}{dt} \right]$$

Applying Laplace transform yields:

$$V_2(s) = \frac{N_2}{N_1} K_v \left[\tau_L s^2 + s \right] \Theta(s)$$

$$\textcircled{2} \quad \frac{\Theta(s)}{V_2(s)} = \frac{(N_1/N_2)}{K_v} \frac{1}{s(\tau_L s + 1)} = \frac{K_m (N_1/N_2)}{s(\tau_L s + 1)}$$

where $\tau_L = \frac{R_A}{K_v K_t} \left(J_m + \left(\frac{N_1}{N_2} \right)^2 J_L \right)$

$$K_m = \frac{1}{K_v}$$

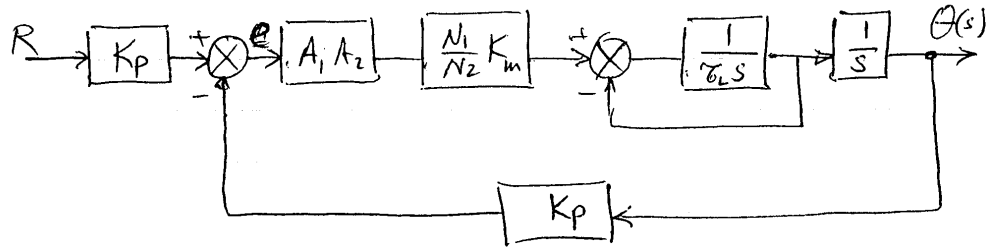
Applying Laplace transform for equation ① yields:

$$\textcircled{3} \quad V_2(s) = A_1 A_2 K_p (R(s) - \Theta(s))$$

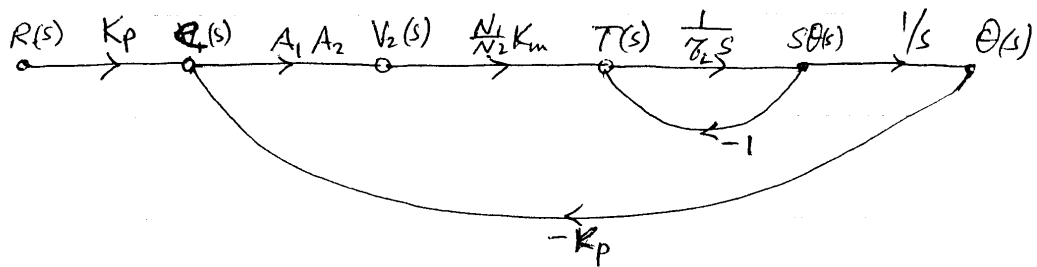
Substituting ③ in ② yields

$$\Theta(s) = \frac{(N_1/N_2) K_m A_1 A_2 K_p [R(s) - \Theta(s)]}{s(\tau_L s + 1)}$$

$$\frac{\Theta(s)}{R(s)} = \frac{(N_1/N_2) K_m A_1 A_2 K_p}{\tau_L s^2 + s + (N_1/N_2) K_m A_1 A_2 K_p}$$



block - diagram .



$$M = \frac{M_1 \Delta_1}{\Delta}$$

$$M_1 = K_p A_1 A_2 \frac{N_1}{N_2} \frac{1}{T_L s} \frac{1}{s} \quad \Delta_1 = 1$$

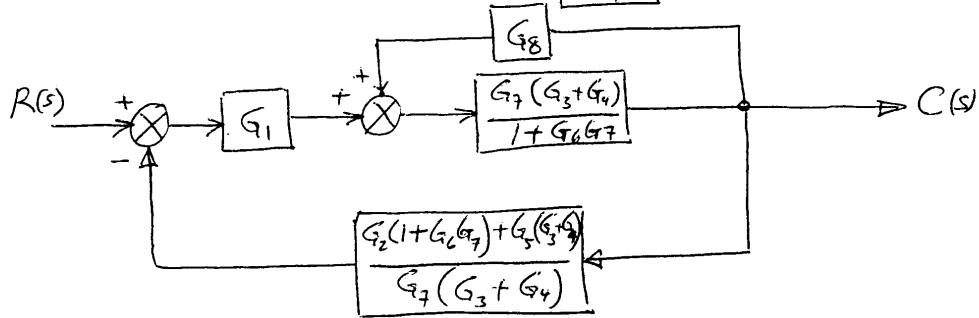
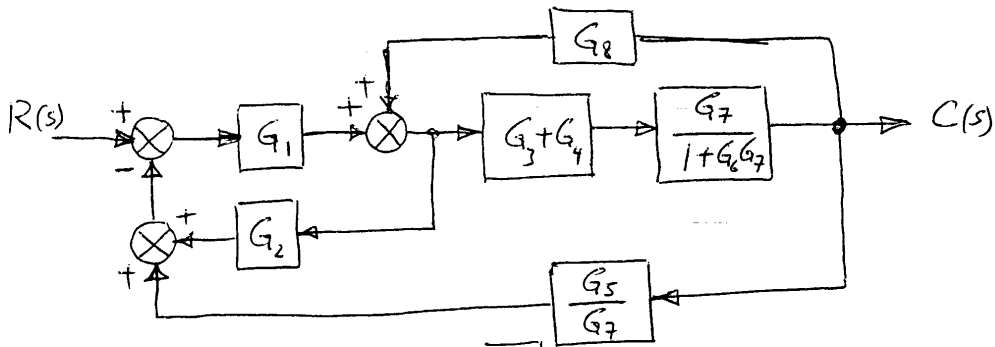
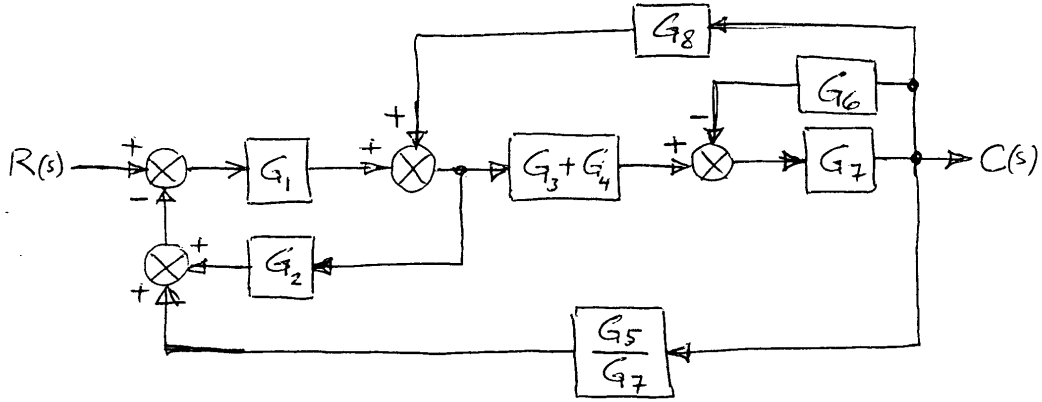
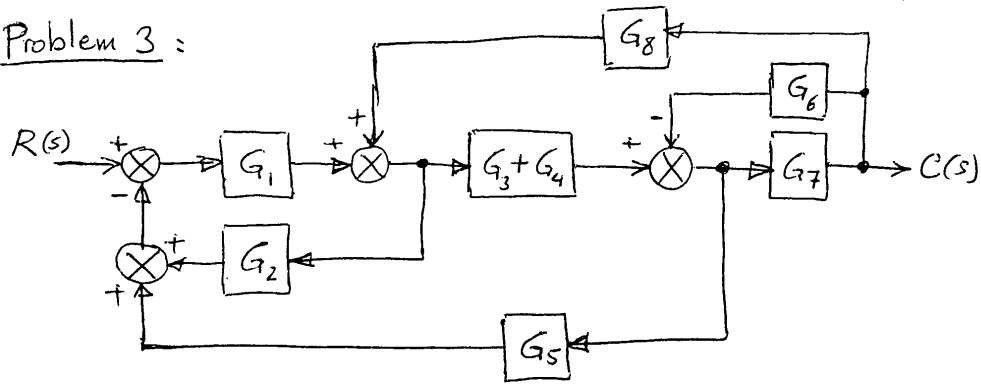
$$L_1 = - \frac{A_1 A_2 (N_1/N_2) K_m K_p}{T_L s^2}$$

$$L_2 = - \frac{1}{T_L s}$$

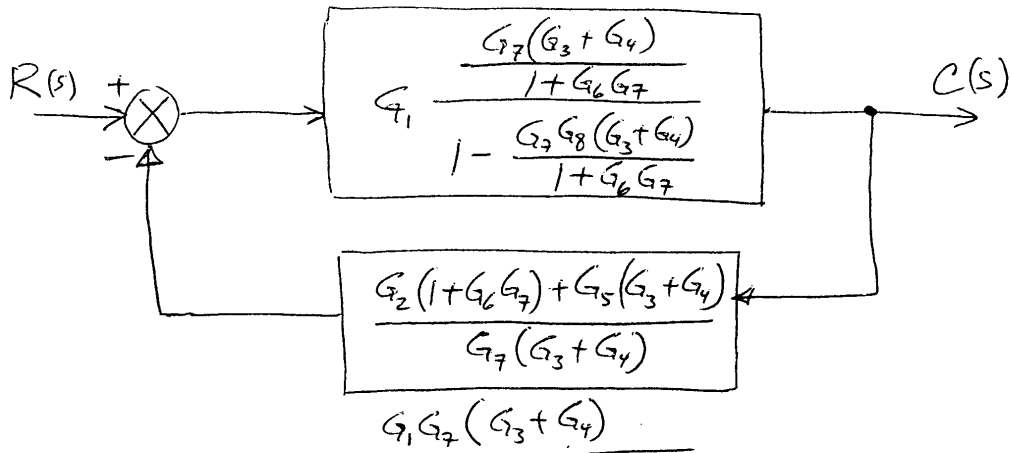
$$M = \frac{K_p A_1 A_2 (N_1/N_2) K_m}{T_L s^2 + s + (N_1/N_2) K_m A_1 A_2 K_p}$$

signal flow graph.

Problem 3 :



Continue problem 2:



$$\frac{C(s)}{R(s)} = \frac{1 + G_6 G_7 - G_7 G_8 (G_3 + G_4)}{1 + \frac{G_1 G_7 (G_3 + G_4)}{1 + G_6 G_7 - G_7 G_8 (G_3 + G_4)} \times \frac{G_2 (1 + G_6 G_7) + G_5 (G_3 + G_4)}{G_7 (G_3 + G_4)}}$$

$$\frac{C(s)}{R(s)} = \frac{G_1 G_7 (G_3 + G_4)}{1 + G_6 G_7 - G_7 G_8 (G_3 + G_4) + G_2 G_1 (1 + G_6 G_7) + G_5 G_1 (G_3 + G_4)}$$

Problem 4:

$$M_1 = G_1 G_2 G_3 G_4 G_5$$

$$\Delta_1 = 1$$

$$M_2 = G_6 G_5$$

$$\Delta_2 = 1 + G_2 H_2 + G_3 H_3$$

$$L_1 = -G_1 H_1$$

$$L_3 L_7 = -G_3 G_5 G_6 H_1 H_3 H_5$$

$$L_2 = -G_2 H_2$$

$$L_1 L_3 = G_1 G_3 H_1 H_3$$

$$L_3 = -G_3 H_3$$

$$L_1 L_4 = G_1 G_4 G_5 H_1 H_4$$

$$L_4 = -G_4 G_5 H_4$$

$$L_1 L_5 = G_1 H_1 H_6$$

$$L_5 = -H_6$$

$$L_2 L_4 = G_2 G_4 G_5 H_2 H_4$$

$$L_6 = -G_2 G_3 G_4 G_5 H_5$$

$$L_2 L_5 = G_2 H_2 H_6$$

$$L_7 = G_6 G_5 H_1 H_5$$

$$L_5 L_3 = G_3 H_3 H_6$$

$$L_8 = G_6 G_5 H_1 H_2 H_3 H_4$$

$$L_1 L_3 L_5 = -G_1 G_3 H_1 H_3 H_6$$

$$\frac{Y_7}{Y_1} = \frac{G_1 G_2 G_3 G_4 G_5 + G_5 G_6 (1 + G_2 H_2 + G_3 H_3)}{1 + G_1 H_1 + G_2 H_2 + G_3 H_3 + G_4 G_5 H_4 + H_6 + G_2 G_3 G_4 G_5 H_5 - G_6 G_5 H_1 H_5 - G_6 G_5 H_1 H_2 H_3 H_4 - G_3 G_5 G_6 H_1 H_3 H_5 + G_1 G_3 H_1 H_3 + G_1 G_4 G_5 H_1 H_4 + G_1 H_1 H_6 + G_2 G_4 G_5 H_2 H_4 + G_2 H_2 H_6 - G_1 G_3 H_1 H_3 H_6 + G_3 H_3 H_6}$$

Problem 5:

$$\Phi(s) = \mathcal{L}^{-1}[(sI-A)^{-1}]$$

$$(sI-A)^{-1} = \begin{bmatrix} s+5 & -1 & 0 \\ 6 & s & -1 \\ 0 & 0 & s \end{bmatrix}^{-1} = \frac{1}{|sI-A|} \begin{bmatrix} s^2 & -6s & 0 \\ s & s(s+5) & 0 \\ 1 & s+5 & s(s+5)+6 \end{bmatrix}^T$$

$$|sI-A| = s(s^2+5s+6) = s(s+2)(s+3)$$

$$(sI-A)^{-1} = \begin{bmatrix} \frac{s}{(s+2)(s+3)} & \frac{-6}{(s+2)(s+3)} & 0 \\ \frac{1}{(s+2)(s+3)} & \frac{(s+5)}{(s+2)(s+3)} & 0 \\ \frac{1}{s(s+2)(s+3)} & \frac{(s+5)}{s(s+2)(s+3)} & \frac{1}{s} \end{bmatrix}^T$$

Characteristic equation

$$s(s+2)(s+3) = 0 \Rightarrow \left. \begin{array}{l} s=0 \\ s=-2 \\ s=-3 \end{array} \right\} \underline{\underline{3 \text{ roots}}}$$

Eigenvalue

$$|\lambda I - A| = \lambda(\lambda+2)(\lambda+3)$$

$\therefore \lambda=0$; $\lambda=-2$, & $\lambda=-3$ are the same the roots of char.

$$\begin{aligned} \frac{s}{(s+2)(s+3)} &= \frac{A_1}{s+2} + \frac{B_1}{s+3} \Rightarrow A_1 = -2 \quad B_1 = +3 \\ \frac{-6}{(s+2)(s+3)} &= \frac{A_2}{s+2} + \frac{B_2}{s+3} \Rightarrow A_2 = -6 \quad B_2 = 6 \\ \frac{1}{(s+2)(s+3)} &= \frac{A_3}{s+2} + \frac{B_3}{s+3} \Rightarrow A_3 = -1 \quad B_3 = +1 \end{aligned}$$

$$\frac{s+5}{(s+2)(s+3)} = \frac{A_4}{s+2} + \frac{B_4}{s+3} \Rightarrow A_4 = 3 \quad B_4 = -2$$

$$\frac{1}{s(s+2)(s+3)} = \frac{A_5}{s} + \frac{B_5}{s+2} + \frac{C_5}{s+3} \Rightarrow A_5 = \frac{1}{6} \quad B_5 = -\frac{1}{2} \quad C_5 = \frac{1}{3}$$

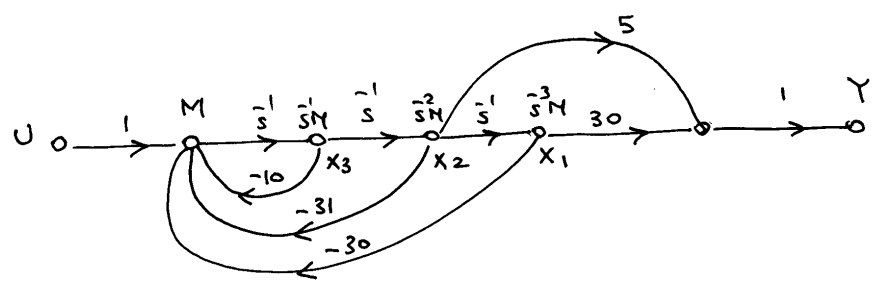
$$\frac{s+5}{s(s+2)(s+3)} = \frac{A_6}{s} + \frac{B_6}{s+2} + \frac{C_6}{s+3} \Rightarrow A_6 = \frac{5}{6} \quad B_6 = -\frac{3}{2} \quad C_6 = \frac{2}{3}$$

$$\phi(t) = \begin{bmatrix} -2t & -3t & -2t & -3t & 0 \\ -2e^{-2t} + 3e^{-3t} & -6e^{-2t} + 6e^{-3t} & 0 \\ -2t & -3t & -2t & -3t & 0 \\ -e^{-2t} + e^{-3t} & 3e^{-2t} - 2e^{-3t} & 0 \\ \frac{1}{6} - \frac{1}{2}e^{-2t} + \frac{1}{3}e^{-3t} & \frac{5}{6} - \frac{3}{2}e^{-2t} + \frac{2}{3}e^{-3t} & 1 \end{bmatrix}^T u(t) \quad t \geq 0$$

(b) Matlab

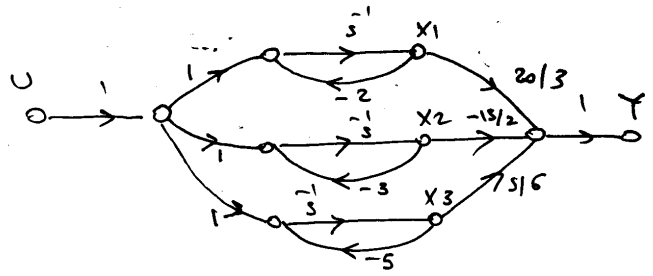
Q.7 $\frac{Y(s)}{U(s)} = \frac{5(s+6)}{s^3 + 10s^2 + 31s + 30}$

a) Phase-Variable format
 $\frac{Y(s)}{U(s)} = \frac{5s^2 + 30s^3}{1 + 10s^{-1} + 31s^{-2} + 30s^{-3}} \cdot \frac{M(s)}{M(s)}$



$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -30 & -31 & -10 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u ; y = [30 \ 5 \ 0] x$$

b) $\frac{Y(s)}{U(s)} = \frac{5(s+6)}{(s+2)(s+3)(s+5)} = \frac{20/3}{s+2} + \frac{-15/2}{s+3} + \frac{5/6}{s+5}$



$$\dot{x} = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -5 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u ; y = \begin{bmatrix} 20/3 & -15/2 & 5/6 \end{bmatrix} x$$