## E 5.8

$$T(s) = \frac{K}{s^2 + \sqrt{2K}\,s + K} \quad ; \quad \omega_n^2 = K \;\&\; 2\zeta\omega_n = \sqrt{2K}$$

$$\therefore \zeta = \frac{1}{\sqrt{2}}$$

a) $$P.O = 100\, e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}\ \% = 100\, e^{\frac{-\pi\sqrt{2}}{\sqrt{\frac{1}{2}}}}\ \% = 4.32\ \%$$

$$\text{Settling Time} = \frac{4}{\zeta\omega_n} = \frac{4\sqrt{2}}{\sqrt{K}} = \frac{8}{\sqrt{2K}}$$

b) $$\frac{8}{\sqrt{2K}} < 1 \longrightarrow K > 32$$

---

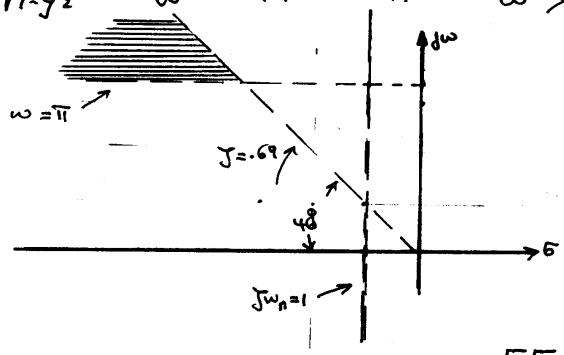
## E 5.9

- $$e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} \leq 0.05$$

$$\sqrt{\frac{(\ln 0.05)^2}{\pi^2 + (\ln 0.05)^2}} \Rightarrow \zeta \geq 0.69$$

- $$\frac{4}{\zeta\omega_n} < 4 \Rightarrow \zeta\omega_n > 1$$

- $$\frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{\omega} < 1 \quad \therefore \quad \omega > \pi$$

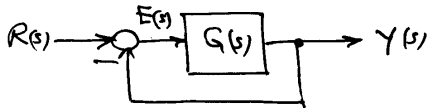


## E 5.10

$$e_{ss} = \lim_{s\to 0} \frac{sR(s)}{1+G(s)} \quad ; \quad G(s) = \frac{10(s+4)}{s(s+1)(s+2)(s+5)}$$

$R(s) = \frac{1}{s}$ : $e(s) = 0$

$R(s) = \frac{1}{s^2}$ : $e(s) = \lim_{s\to 0} \frac{1}{s + \frac{10(s+4)}{(s+1)(s+2)(s+5)}} = \frac{1}{4}$




(Note that $K_p = \infty$ & $K_v = 4$)

---

## E 5.12

$$G(s) = \frac{10}{s^2 + 14s + 50}$$

System is type 0 $\therefore$ $K_p = \lim_{s\to 0} G(s) = 1/5 = 0.2$

- for a step $e_{ss} = \frac{1}{1+K_p} = \frac{1}{1.2} = 0.833$
- for a ramp $K_v = \lim_{s\to 0} sG(s) = 0$

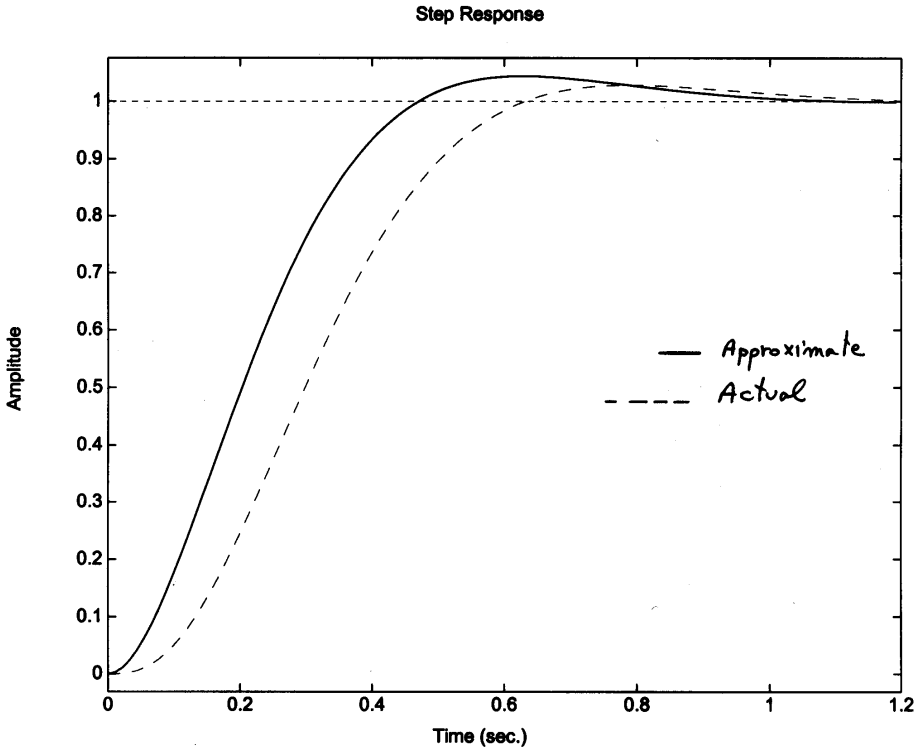
$$e_{ss} = \frac{1}{K_v} = \infty$$

---

## E 5.14

a) plot of $T(s) = \dfrac{500}{(s+10)(s^2+10s+50)}$ (--- dashed)

b) plot of $T_1(s) = \dfrac{50}{(s^2+10s+50)}$ ; Approximate; (— solid)

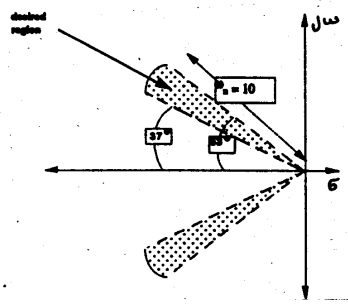



Note before neglecting the pole at $s=-10$; the T.F. $T(s)$ must be expressed as

$$\frac{500}{10\left(\frac{1}{10}s+1\right)(s^2+10s+50)} \simeq \frac{50}{s^2+10s+50}$$

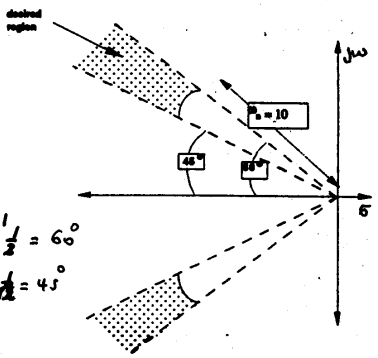
# E 5.16

a) $0.6 \le \zeta \le 0.8$ ; $\omega_n \le 10$




Note: $\cos^{-1} 0.6 = 63^\circ$ ; $\cos^{-1} 0.8 = 37^\circ$

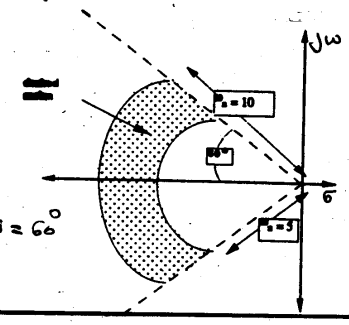
b) $1/2 \le \zeta \le 0.707$ ; $\omega_n \ge 10$




$\cos^{-1} \frac{1}{2} = 60^\circ$

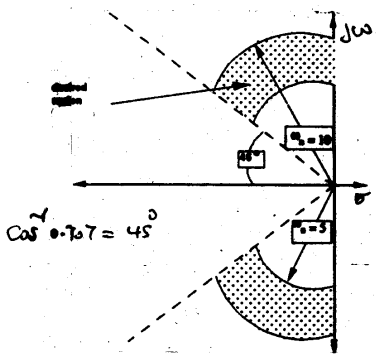
$\cos^{-1} \frac{1}{\sqrt{2}} = 45^\circ$

c) $\zeta \ge 0.5$ ; $5 \le \omega_n \le 10$



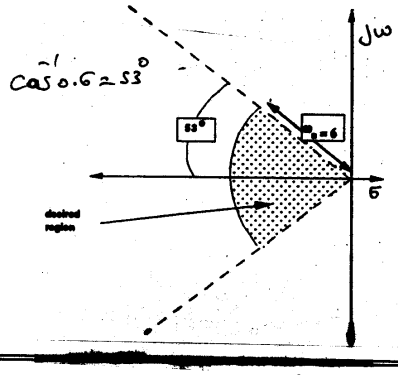

$\cos^{-1} 0.5 = 60^\circ$

d) $\zeta \le 0.707$ ; $5 \le \omega_n \le 10$



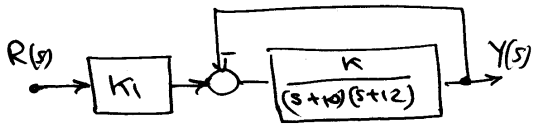

$\cos^{-1} 0.707 = 45^\circ$

e) $\zeta \ge 0.6$ , $\omega_n \le 6$




$\cos^{-1} 0.6 = 53^\circ$

# P 5.19




a) $E(s) = R(s) - Y(s)$

$$E(s) = R(s)\left[1 - \frac{KK_1}{(s+10)(s+12)+K}\right]$$

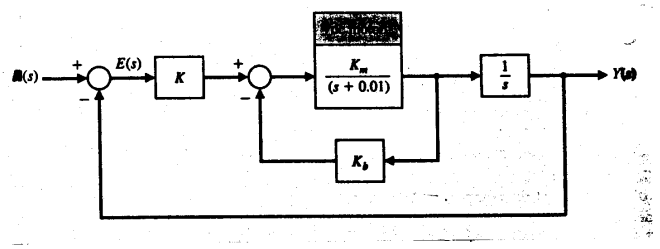
$$E(s) = \frac{(s+10)(s+12)+K(1-K_1)}{(s+10)(s+12)+K} R(s) \quad ; \quad R(s) = \frac{1}{s}$$

$$e_{ss} = \lim_{s \to 0} sE(s) = \frac{120 + K - KK_1}{120 + K}$$

b) for $e_{ss} = 0 \Rightarrow$ set $120 + K - KK_1 = 0$

$$\therefore K_1 = 1 + \frac{120}{K}$$

# AP 5.6



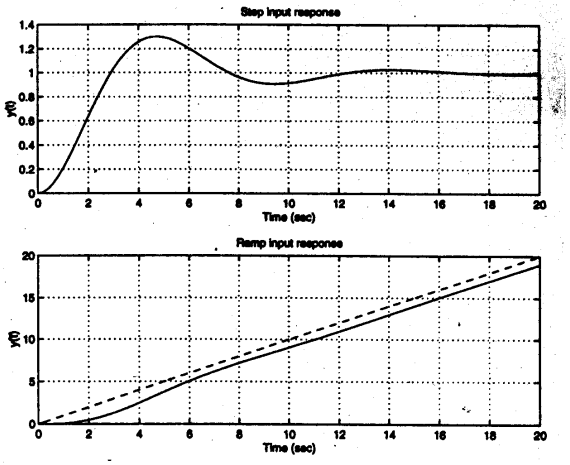

a) $$\frac{E(s)}{R(s)} = \frac{1 \cdot \left[1 + \frac{K_m K_b}{s+0.01}\right]}{1 - \left[-\frac{K_m K_b}{s+0.01} - \frac{K K_m}{s(s+0.01)}\right]}$$

$$E(s) = \frac{s(s + 0.01 + K_m K_b)}{K K_m + s(K_m K_b + 0.01) + s^2} R(s)$$

for $R(s) = \frac{1}{s^2}$ $\quad e_{ss} = \lim_{s \to 0} s E(s) = \frac{K_m K_b + 0.01}{K K_m}$

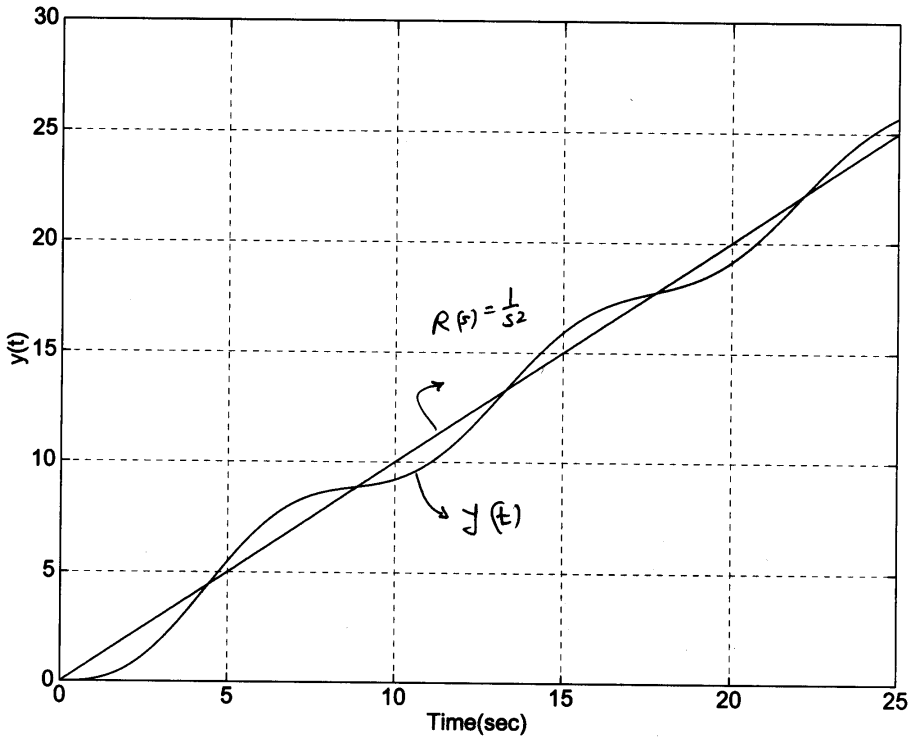
b) $10 \times 0.05 + 0.01 = 10K \Rightarrow K = \frac{0.51}{10} = 0.051$

c)



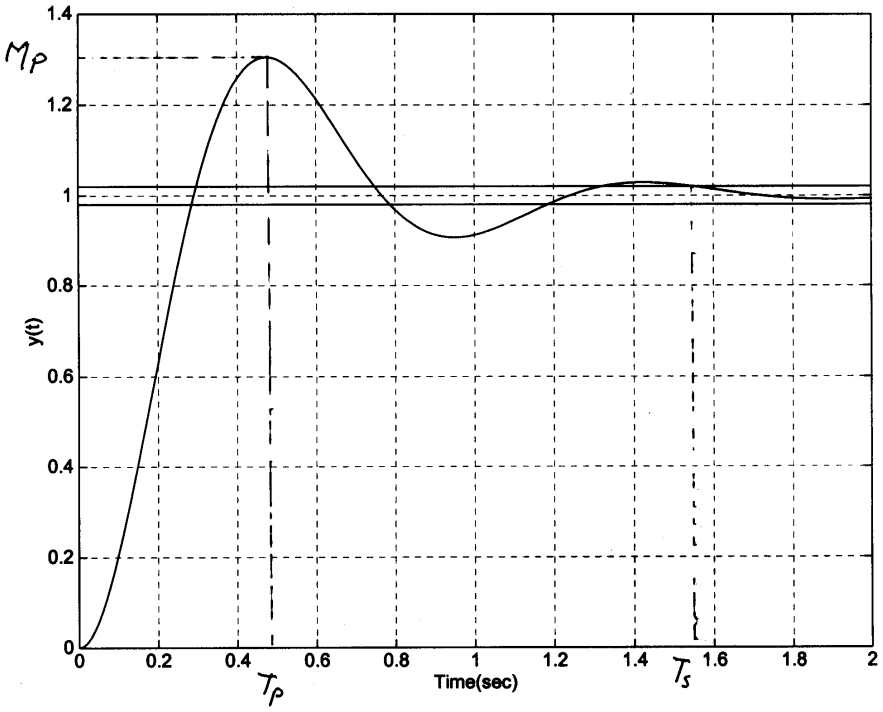
MP 5.2

```
t = 0.0:0.01:25; u = t;
[Num,Den]= cLoop([1 5],[1 10 0 0]);
Lsim(Num, Den, u, t); Hold; Plot(t, t); Grid
```



MP 5.5

```
[num, den] = cloop([50];[1 50]);
t = 0:0.01:2.0 ;[y,x] = step(num, den, t)
plot(t, y, [0 2], [.98 .98], [0 2], [1.02 1.02]); Grid
xLabel('Time (Sec)') ; yLabel('y(t)')
```




$M_p = 1.3$ ; $T_p = 0.47$ ; $T_s = 1.55$

HW #4

EE 380 1981