

E 5.8

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$$T(s) = \frac{K}{s^2 + \sqrt{2K}s + K} \quad ; \quad \omega_n^2 = K \quad \& \quad 2J\omega_n = \sqrt{2K}$$

$$\begin{aligned} \text{a) P.O.} &= 100 e^{\frac{-\pi J}{\sqrt{1-J^2}}} \% \\ &= 100 e^{\frac{-\pi \sqrt{2}}{2}} \% = 4.32 \% \end{aligned}$$

$$\text{Settling Time} = \frac{4}{J\omega_n} = \frac{4\sqrt{2}}{\sqrt{K}} = \frac{8}{\sqrt{2K}}$$

$$\text{b) } \frac{8}{\sqrt{2K}} < 1 \quad \rightarrow \quad K > 32$$

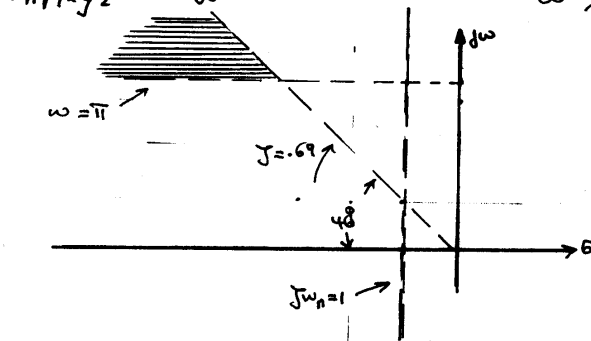
E 5.9

$$e^{\frac{-\pi J}{\sqrt{1-J^2}}} \leq 0.05$$

$$\sqrt{\frac{(\ln 0.05)^2}{\pi^2 + (\ln 0.05)^2}} \Rightarrow J \geq 0.69$$

$$\frac{4}{J\omega_n} < 4 \Rightarrow J\omega_n > 1$$

$$\frac{\pi}{\omega_n \sqrt{1-J^2}} = \frac{\pi}{\omega} < 1 \quad \therefore \omega > \pi$$



HW #4

EE 380/981

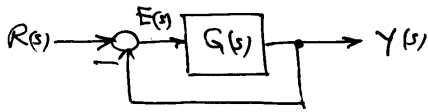
E 5.10

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$$e_{ss} = \lim_{s \rightarrow 0} \frac{sR(s)}{1+G(s)} \quad ; \quad G(s) = \frac{10(s+4)}{s(s+1)(s+2)(s+5)}$$

$$R(s) = \frac{1}{s} \quad ; \quad e(s) = 0$$

$$R(s) = \frac{1}{s^2} \quad ; \quad e(s) = \lim_{s \rightarrow 0} \frac{1}{s + \frac{10(s+4)}{(s+1)(s+2)(s+5)}} = \frac{1}{4}$$



(Note that $k_p = \infty$ & $k_v = 4$)

E 5.12

$$G(s) = \frac{10}{s^2 + 14s + 50}$$

System is type 0 $\therefore k_p = \lim_{s \rightarrow 0} G(s) = 1/5 = 0.2$

• for a step $e_{ss} = \frac{1}{1+k_p} = \frac{1}{1.2} = 0.833$

• for a ramp $k_v = \lim_{s \rightarrow 0} s G(s) = 0$

$$e_{ss} = \frac{1}{k_v} = \infty$$

HW #4

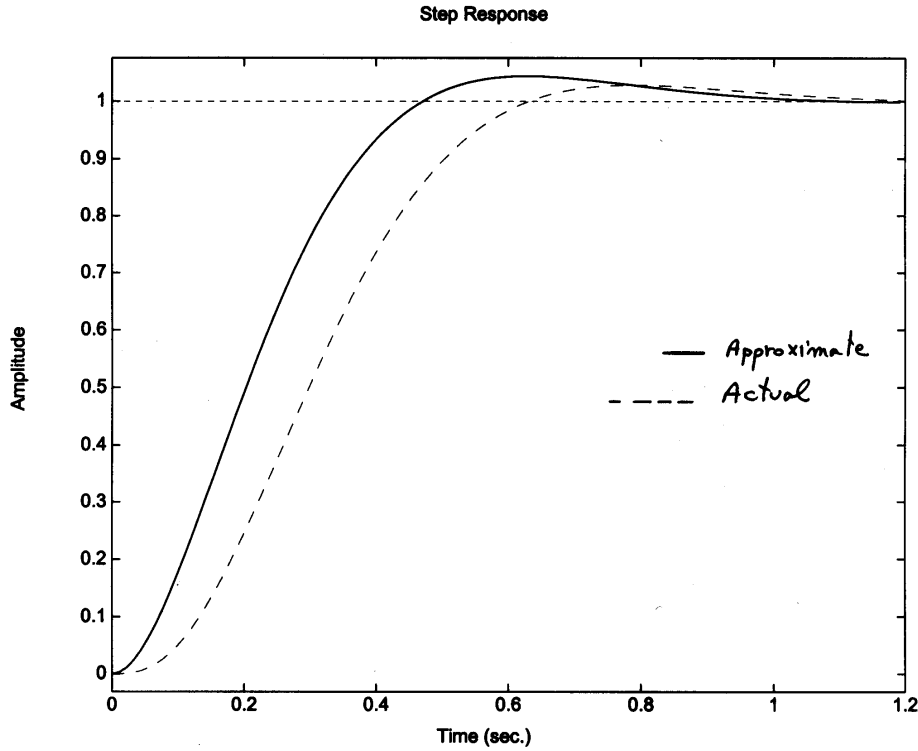
EE 380/981

E 5.14

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a) plot of $T(s) = \frac{500}{(s+10)(s^2+10s+50)}$ (--- dashed)

b) plot of $T_1(s) = \frac{50}{(s^2+10s+50)}$; Approximate; (— solid)



Note before neglecting the pole at $s=-10$; the T.F. $T(s)$ must be expressed as

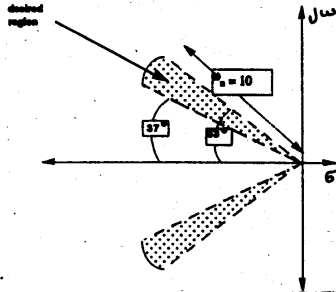
$$\frac{500}{10\left(\frac{1}{10}s+1\right)(s^2+10s+50)} \approx \frac{50}{s^2+10s+50}$$

HW #4

EE 380/981

E 5.16

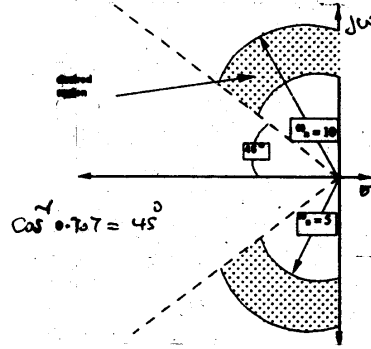
a) $0.6 \leq \zeta \leq 0.8$; $\omega_n \leq 10$



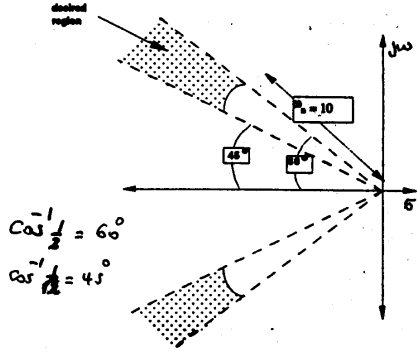
Note: $\cos^{-1} 0.6 = 63^\circ$; $\cos^{-1} 0.8 = 37^\circ$

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d) $\zeta \leq 0.707$; $5 \leq \omega_n \leq 10$

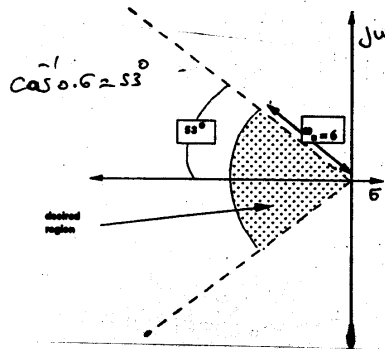


b) $1/2 \leq \zeta \leq 0.707$; $\omega_n \geq 10$



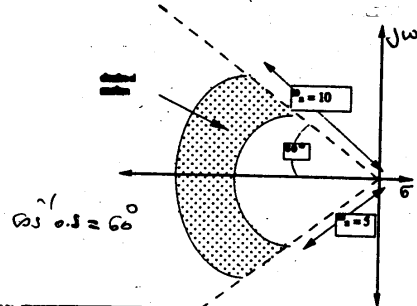
$\cos^{-1} 1/2 = 60^\circ$
 $\cos^{-1} 1/2 = 45^\circ$

e) $\zeta \geq 0.6$, $\omega_n \leq 6$



$\cos^{-1} 0.6 = 53^\circ$

c) $\zeta \geq 0.5$; $5 \leq \omega_n \leq 10$

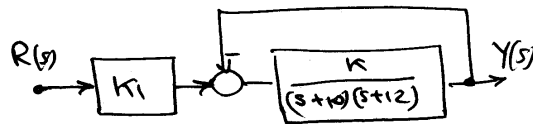


$\cos^{-1} 0.5 = 60^\circ$

Hw #4

EE 380/981

P 5.19



a) $E(s) = R(s) - Y(s)$

$$E(s) = R(s) \left[1 - \frac{K K_i}{(s+10)(s+12) + K} \right]$$

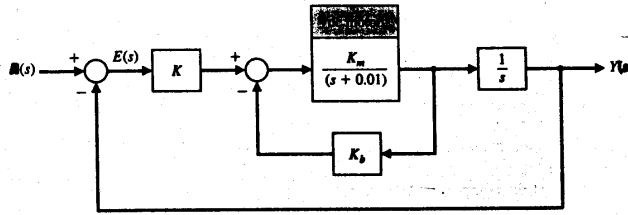
$$E(s) = \frac{(s+10)(s+12) + K(1-K_i)}{(s+10)(s+12) + K} R(s) \quad ; \quad R(s) = \frac{1}{s}$$

$$e_{ss} = \lim_{s \rightarrow 0} s E(s) = \frac{120 + K - K K_i}{120 + K}$$

b) for $e_{ss} = 0 \implies$ set $120 + K - K K_i = 0$
 $\therefore K_i = 1 + \frac{120}{K}$

AP 5.6

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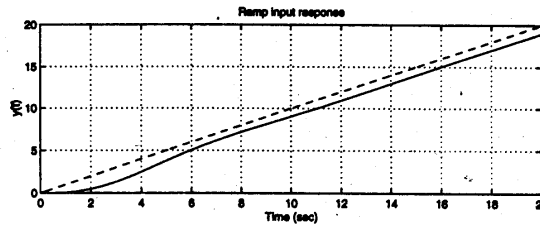
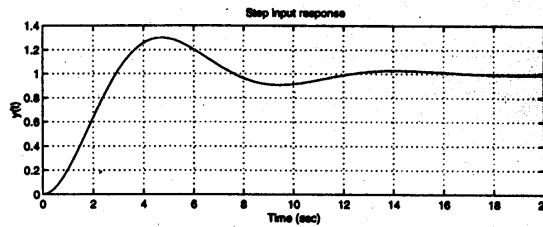
$$a) \quad \frac{E(s)}{R(s)} = \frac{1 \cdot \left[1 + \frac{k_m k_b}{s+0.01} \right]}{1 - \left[-\frac{k_m k_b}{s+0.01} - \frac{k k_m}{s(s+0.01)} \right]}$$

$$E(s) = \frac{s(s+0.01+k_m k_b)}{k k_m + s(k_m k_b + 0.01) + s^2} R(s)$$

for $R(s) = \frac{1}{s^2}$ $e_{ss} = \lim_{s \rightarrow 0} s E(s) = \frac{k_m k_b + 0.01}{k k_m}$

b) $10 \times 0.05 + 0.01 = 10k \Rightarrow k = \frac{0.51}{10} = 0.051$

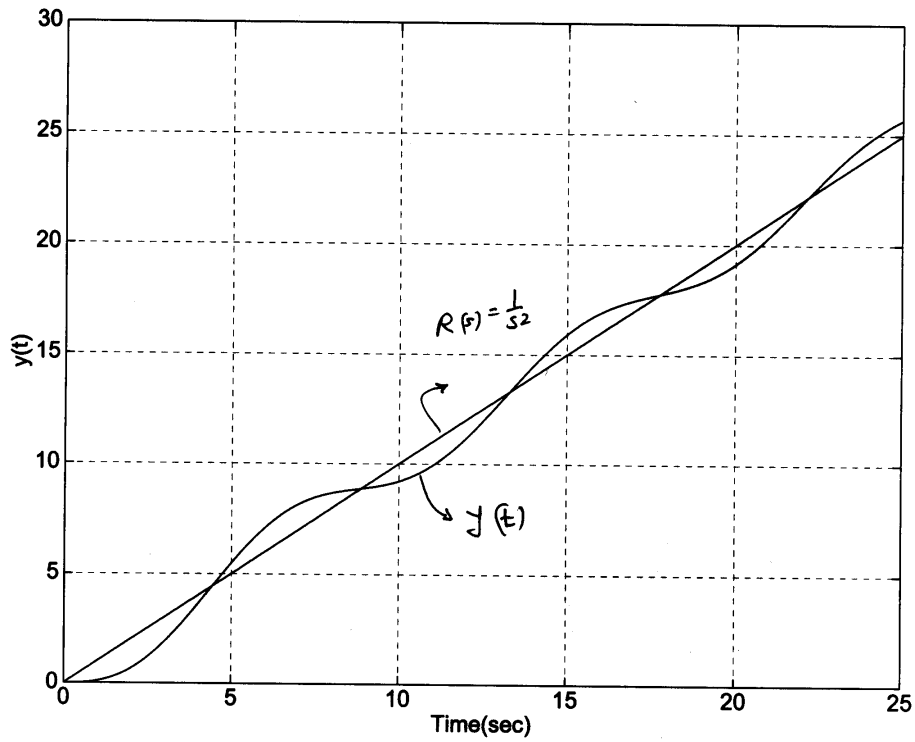
c)



MP 5.2

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```
t = 0:0.01:25; u = t;  
[Num, Den] = cLoop([1 5], [1 10 0 0]);  
LsM(Num, Den, u, t); Hold; Plot(t, t); Grid
```



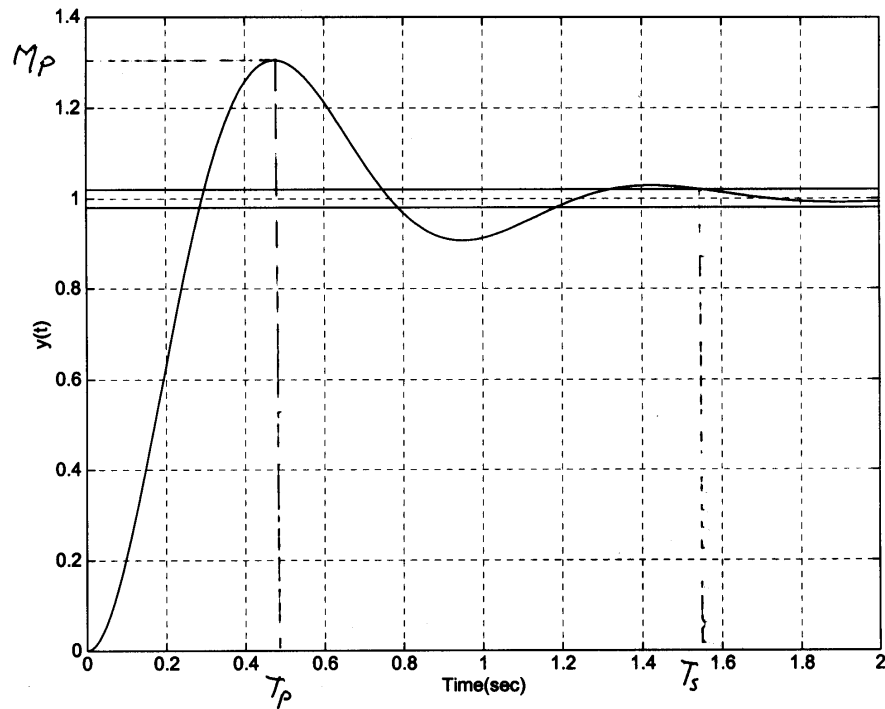
HW # 4

EE 380/981

MP 5.5

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```
[num, den] = cloop([5 0]; [1 5 0]);  
t = 0:0.01:2.0; [y, x] = step(num, den, t);  
plot(t, y, [0 2], [0.98 1.02], [0 2], [1.02 1.02]), Grid  
xlabel('Time (sec)'); ylabel('y(t)')
```



$M_p = 1.3$; $T_p = 0.47$; $T_s = 1.55$

Hw # 4

EE 380 / 981