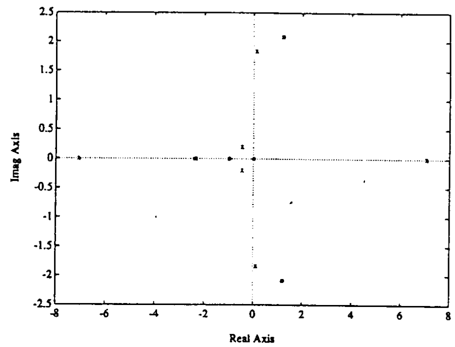
MP2.6 The pole-zero map is shown in Figure MP2.6.




```
% Part (a)
num1=[4]; den1=[1];
num2=[1]; den2=[1 1];
num3=[1 0]; den3=[1 0 2];
num4=[1]; den4=[1 0 0];
num5=[4 2]; den5=[1 2 1];
num6=[50]; den6=[1];
num7=[1 0 2]; den7=[1 0 0 14];
[numa,dena]=feedback(num4,den4,num6,den6,+1);
numb=conv(num5,dena); denb=conv(den5,numa);
[numc,denc]=parallel(numb,denb,num7,den7);
[numd,dend]=series(num2,den2,num3,den3);
[nume,dene]=series(numd,dend,numa,dena);
[numf,denf]=feedback(nume,dene,numc,denc);
[num,den]=series(num1,den1,numf,denf);
printsys(num,den)
% Part (b)
pzmap(num,den)
% Part (c)
p=roots(den)
z=roots(num)
```

poles

zeros

```
p =

  7.0709
 -7.0713
  1.2051 + 2.0863i
  1.2051 - 2.0863i
  0.1219 + 1.8374i
  0.1219 - 1.8374i
 -2.3933
 -2.3333
 -0.4635 + 0.1997i
 -0.4635 - 0.1997i

z =
      0
  1.2051 + 2.0872i
  1.2051 - 2.0872i
 -2.4101
 -1.0000 + 0.0000i
 -1.0000 - 0.0000i
```

FIGURE MP2.6
Pole-zero map.

The closed-loop transfer function is

$$T(s) = \frac{4s^6 + 8s^5 + 4s^4 + 56s^3 + 112s^2 + 56s}{\Delta(s)},$$

where

$$\Delta(s) = s^{10} + 3s^9 - 45s^8 - 125s^7 - 200s^6 - 1177s^5 - 2344s^4 - 3485s^3 - 7668s^2 - 5598s - 1400\,.$$

Problem 1

(a) $M_1 = G_1 G_2 G_3 G_4 \qquad \Delta_1 = 1$

$M_2 = G_3 G_4 G_5 \qquad \Delta_2 = 1$

$L_1 = -G_2 H_1 \qquad L_1 L_2 = G_2 G_4 H_1 H_2$

$L_2 = -G_4 H_2$

$L_3 = -G_1 G_2 G_3 G_4 H_3$

$L_4 = -G_3 G_4 G_5 H_3$

$$\left.\frac{Y_6}{Y_1}\right|_{Y_7=0} = \frac{G_1 G_2 G_3 G_4 + G_3 G_4 G_5}{1 + G_2 H_1 + G_4 H_2 + G_1 G_2 G_3 G_4 H_3 + G_3 G_4 G_5 H_3 + G_2 G_4 H_1 H_2}$$

$$\left.\frac{Y_6}{Y_7}\right|_{Y_1=0} = \frac{1 + G_2 H_1}{1 + G_2 H_1 + G_4 H_2 + G_1 G_2 G_3 G_4 H_3 + G_3 G_4 G_5 H_3 + G_2 G_4 H_1 H_2}$$

(b) $M_1 = G_1 G_2 G_3 G_4 \qquad \Delta_1 = 1$

$M_2 = G_3 G_4 G_5 \qquad \Delta_2 = 1$

$L_1 = -G_1 H_1 \qquad L_1 L_2 = G_1 G_3 H_1 H_2$

$L_2 = -G_3 H_2 \qquad L_1 L_3 = G_1 G_3 G_4 H_1 H_3$

$L_3 = -G_3 G_4 H_3$

$L_4 = -G_1 G_2 G_3 G_4 H_4$

$L_5 = -G_3 G_4 G_5 H_3$

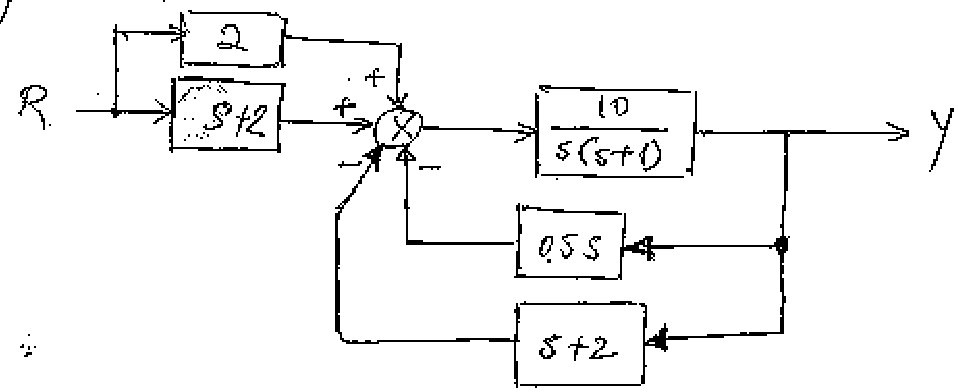
$$\Delta = 1 + G_1 H_1 + G_3 H_2 + G_3 G_4 H_3 + G_1 G_2 G_3 G_4 H_4 + G_3 G_4 G_5 H_3 + G_1 G_3 H_1 H_2 + G_1 G_3 G_4 H_1 H_3$$

$$\left.\frac{Y_6}{Y_1}\right|_{Y_7=0} = \frac{G_1 G_2 G_3 G_4 + G_3 G_4 G_5}{\Delta}$$

$$\left.\frac{Y_6}{Y_7}\right|_{Y_1=0} = \frac{1 + G_1 H_1 + G_3 H_2 + G_1 G_3 H_1 H_2}{\Delta}$$

Problem 2 : $\left.\frac{Y(s)}{R(s)}\right|_{N=0} = ?$

ⓐ



$$\frac{Y(s)}{R(s)} = (s+4)\,\frac{\frac{10}{s(s+1)}}{1+\frac{10(1.5s+2)}{s(s+1)}} = \frac{10(s+4)}{s^2+s+15s+20}$$

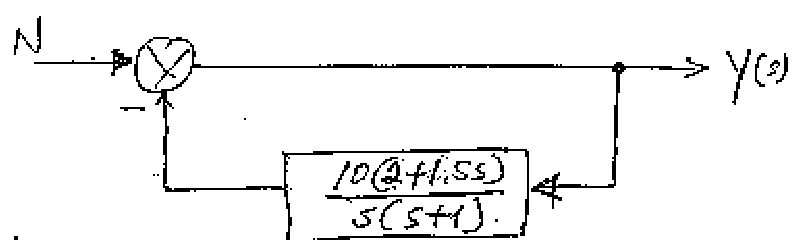
$$\left.\frac{Y(s)}{R(s)}\right|_{N=0} = \frac{10(s+4)}{s^2+16s+20}$$

ⓑ $R(s)-Y(s)=E(s) \Rightarrow \frac{Y(s)}{G(s)}-Y(s)=E(s)$

$$\frac{Y(s)}{E(s)} = \frac{G(s)}{1-G(s)} = \frac{\frac{10(s+4)}{s^2+16s+20}}{1-\frac{10(s+4)}{s^2+16s+20}}$$

$$\frac{Y(s)}{E(s)} = \frac{10(s+4)}{s^2+6s-20}$$

ⓒ



$$\left.\frac{Y(s)}{N(s)}\right|_{R=0} = \frac{1}{1+\frac{10(1.5s+2)}{s(s+1)}} = \frac{s(s+1)}{s^2+16s+20}$$

Problem 3

(b) SFG $N=0$

$M_1 = G_1 G_2 G_3 \qquad \Delta_1 = 1$

$M_2 = G_4 \qquad \Delta_2 = 1$

$L_1 = -G_1 G_2 H_1 \qquad L_5 = G_2 G_4 H_1 H_2$

$L_2 = -G_2 G_3 H_2$

$L_3 = -G_1 G_2 G_3$

$L_4 = -G_4$

$$\left.\frac{Y(s)}{R(s)}\right|_{N=0} = \frac{G_1 G_2 G_3 + G_4}{1 + G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 + G_4 - G_2 G_4 H_1 H_2}$$

$R=0$

$M_1 = 1 \qquad \Delta_1 = 1 + G_1 G_2 H_1$

$$\left.\frac{Y(s)}{N(s)}\right|_{R=0} = \frac{1 + G_1 G_2 H_1}{1 + G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 + G_4 - G_2 G_4 H_1 H_2}$$

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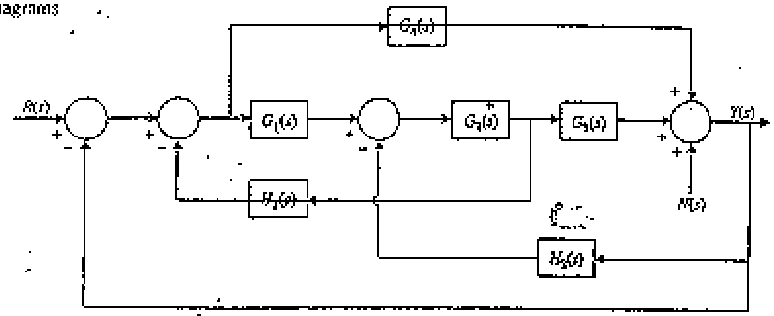
Problem 3:

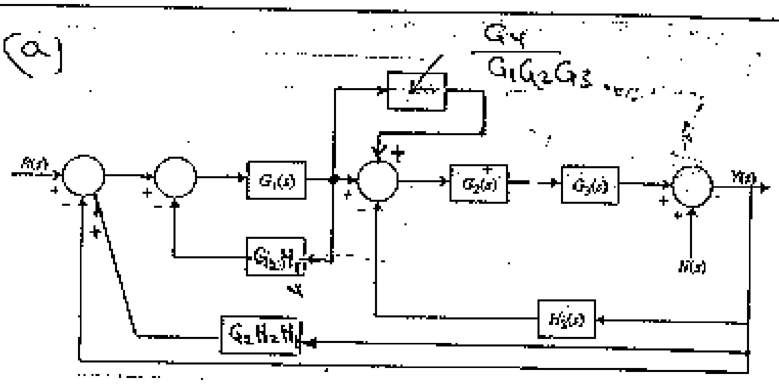
Find the following transfer functions: $a)\left.\dfrac{Y(s)}{R(s)}\right|_{N=0}$ $\quad b)\left.\dfrac{Y(s)}{N(s)}\right|_{R=0}$

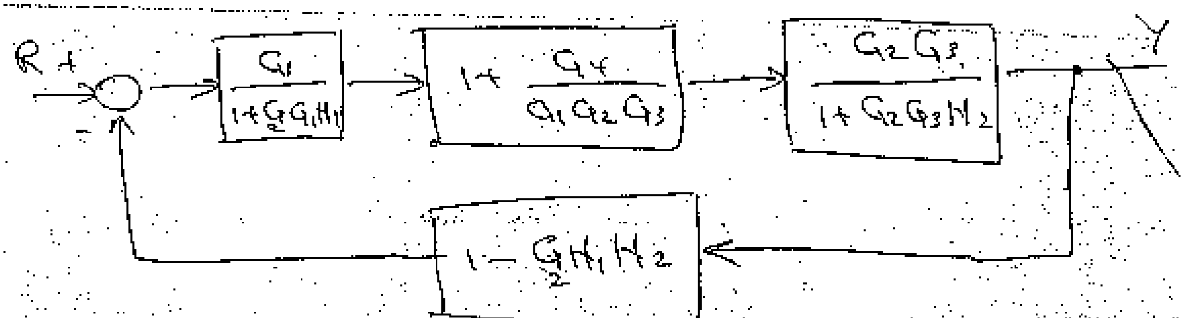
diagrams



(a) Using block diagram reduction method.
(b) Applying the SFG gain formula directly to block diagram to find the transfer functions.

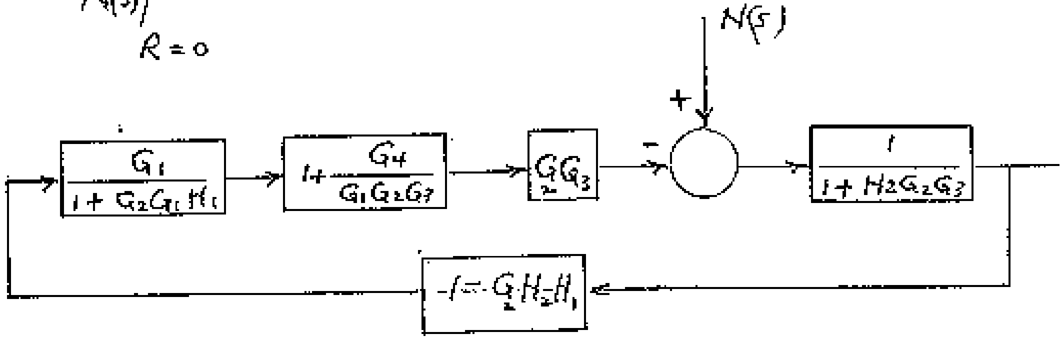
(a)





$$\frac{Y(s)}{R(s)} = \frac{G_1G_2G_3 + G_4}{1 + G_1G_3H_2 + G_1G_2H_1 + G_1G_2G_3 + G_4 - G_4G_2H_2H_1}$$

b) $\left.\dfrac{Y(s)}{N(s)}\right|_{R=0}$



$$\left.\frac{Y(s)}{N(s)}\right|_{R=0} = \frac{1 \Big/ (1+G_3G_2H_2)}{\dfrac{(1+G_1G_2H_1)(1+H_2G_2G_3) + (1-G_2H_2H_1)(G_1G_2G_3+G_4)}{(1+H_2G_2G_3)(1+G_1G_2H_1)}}$$

$$\left.\frac{Y(s)}{N(s)}\right|_{R=0} = \frac{1+G_1G_2H_1}{1+G_2G_3H_2+G_1G_2H_1+G_1G_2G_3+G_4-G_2G_4H_1H_2}$$