Problem 11-1

$R = 5\ \Omega$, $V_s = 120$ V, $V_m = \sqrt{2} \times 120 = 169.7$ V, f = 60 Hz

$k = n/(n+m) = 125/(125+75) = 0.625$

(a) From Eq. (11-1), $V_o = V_s \sqrt{k} = 120 \sqrt{0.625} = 94.87$ V

The rms load current is $I_o = V_o/R = 94.87/5 = 18.97$ A

(b) The load power is $P_o = I_o^2\ R = 18.972 \times 5 = 1800$ W

$VA = V_s I_s = V_s I_o = 120 \times 18.97 = 2276.4$ W

$PF = P_o/VA = \sqrt{k} = 0.791$

(c) The peak thyristor current is $I_m = V_m/R = 169.7/5 = 33.9$ A

From Eq. (11-3), the average thyristor current is

$I_A = (33.9/\pi) \times 0.625 = 6.74$ A

$I_R = 33.9 \times \sqrt{0.625/2} = 13.42$ A

Problem 11-3

$R = 5\ \Omega$, $V_s = 120$ V, $\alpha = \pi/3$, and $V_m = \sqrt{2} \times 120 = 169.7$ V

(a) From Eq. (11-5), $V_o = 120\ [(2\pi - \alpha + \sin 2\alpha/2)/2\pi]^{1/2} = 120 \times 0.9498 =$ 113.98 V

(b) $I_o = V_o/R = 113.9/5 = 22.8$ A

$P_o = I_o^2\ R = 22.82 \times 5 = 2598.5$ W

$VA = V_s I_s = V_s I_o = 120 \times 22.98 = 2736$ W

$PF = P_o/VA = 2598.5/2736 = 0.9498$

(c) From Eq. (11-6), the average output voltage is

$V_{dc} = \sqrt{2} \times 120\ (\cos \pi/3 - 1)/2\pi = -\ 13.51$ V

The average input current is $I_D = V_{dc}/R = -\ 13.51/5 = -\ 2.7$ A

Problem 11-6

R = 1.5 Ω, V_s = 120 V, P_o = 7500 W, $V_m = \sqrt{2} \times 120 = 169.7$ V

(a) Since $P_o = V_o^2 / R$, Eq. (11-8) gives

$$P_o = 2000 = \frac{V_o^2}{R} = \frac{V_S^2}{R} = \left[\frac{1}{2\pi}\left(2\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]$$

By iteration yields, α = 62.74o

(b) From Eq. (11-8),

$$V_o = V_S\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{1/2} = 106.1 \text{ V}$$

(c) $P_o = V_o^2 / R$ = 106.12 /1.5 = 7498 W

VA = $V_s I_s = V_s I_o$ = 120 x 106.1/1.5 = 8520

PF = 7498/8520 = 0.88

(d) From Eq. (11-10), $I_A = \sqrt{2} V_s (\cos \alpha + 1)/(2\pi R)$ = 26.25 A

(e) $I_R = I_o / \sqrt{2}$ = (106.1/1.5)/ √2 = 49.99 A

Problem 11-8

R = 5 Ω, L = 5 mH, f = 60 Hz, ω = 2 π x 60 = 377 rad/s, V_s = 120 V, α = 60° and ϕ = 20.66°

(a) The extinction angle can be determined from the solution of Eq. (11-16) and an iterative solution (by Mathcad software) yields ß = 200.66°. The conduction angle is δ = ß - α = 200.66 - 60 = 140.66°.

(b) From Eq. (11-18), V_o = 108.3 V

(c) By numerical integration of Eq. (11-19) between the limits, ω t = α to ß, gives the rms thyristor current as I_R = 13.65 A.

(d) From Eq. (11-20), $I_o = \sqrt{2} \times 13.65$ = 19.3 A.

(e) Numerical integration of Eq. (11-21) yields the average thyristor current as I_A = 7.76 A

(f) $P_o = 19.3^2 \times 5$ = 1863.4 W and VA = 120 x 19.3 = 2316 W

PF = P_o/VA = 1863.4/2316 = 0.8045 (lagging)

Problem 11-9

V_s = 120 V, V_m = √2 x 120 = 169.7 V

(a) R = 5 Ω, L = 5 mH

The Mathcad software is used to calculate the input power factor for various values of delay angle.

α°	β	VO	PO	PF
0	200.655	120.5798	2583.717	.9471663
15	200.655	120.3553	2521.621	.9357152
30	200.655	118.8456	2518.951	.9352196
45	200.655	115.0275	2479.952	.9279518
60	200.655	108.2787	2359.558	.9051469
75	270.1568	129.5029	2130.412	.8600734
90	270.1568	120.1045	1802.817	.7911879
105	199.905	70.56311	1411.449	.7000618
120	199.155	54.10058	995.3172	.5878745
135	202.905	38.69433	613.9442	.4617089
150	195.1551	21.70204	313.5677	.3299662
165	190.1551	8.42564	115.4636	.2002287

(b) R = 5 Ω, L = 0 mH. With a resistive load, the power factor can be found rom Eq. (11-9) for various values of delay angles as follows:

Delay angle	Full-wave pf
0	1
15	.9981203
30	.9854773
45	.9534961
60	.8969386
75	.8141933
90	.7071067
105	.5805938
120	.4421551
135	.3014052
150	.1698073
165	6.128576E-02
180	3.220536E-04

Problem 11-28

$\alpha = \pi/3$, L = 6.5 mH, R = 5 Ω, = 2π x 60 = 377 rad/s and V_s = 208 V

From Eq. (11-54)

$$i_o(t) = \sum_{n=1,3,5,.}^{\infty} \sqrt{2}\, I_n \sin(n\omega t + \phi_n - \varphi_n)$$

where $\phi_n = \tan^{-1}(n\omega L/R)$, $\varphi_n = \tan^{-1}(a_n/b_n)$,

and $I_n = \dfrac{\sqrt{a_n^2 + b_n^2}}{\sqrt{2} \times \sqrt{R^2 + (n\omega L)^2}}$

From Eqs. (11-50) and (11-51), the Fourier coefficients a_n and b_n are calculated as,

N	A(N)	B(N)	V(N)	PHI(N)	THETA(N)
1	22.5134	402.004	402.6339	3.205383	26.10894
3	-55.85325	-78.86111	96.63675	35.30797	55.77845
5	61.93438	37.85704	72.58804	58.56488	67.80014
7	-42.73995	-6.772272	43.27317	80.99616	73.74899
9	14.68604	-2.214494	14.85207	-81.42503	77.22604
11	4.253526		-6.862211	8.073563	-31.7926
79.49135					
13	-6.809434		19.54615	20.69831	-19.20718
81.07976					
15	-.9352256		-23.2668	23.28559	2.301806
82.25361					

Note: The angle of PHI(N) depends on the signs of A(N) and B(N)

PEAK VALUE OF NTH HARMONIC CURRENTS

1	72.30975
3	10.8696
5	5.485311
7	2.421964
9	.6567738
11	.2944977
13	.6418926
15	.6277258

Possible reactances of the capacitor = -.4638901 .4270413

RMS value of thrid harmonic current = 7.685964

Filter capacitance in µF = -1906.042 , or 2070.512

From Example 11-4, we get the extinction angle as ß = 220.35°. For known values of α, ß, R, L and V_s, a_n and b_n of the Fourier series in Eq. (11-53) and the load current i_o of Eq. (11-54) can be calculated. The load current is given by

$i_o(t) = 72.3 \sin(\omega t - 26.1° - 3.2°)$

$+ 10.87 \sin(3\omega t - 55.78° - 215.31°)$

$+ 5.49 \sin(5\omega t - 67.8° - 58.56°)$

$+ 2.42 \sin(7\omega t - 73.75° - 261°)$

$+ 0.66 \sin(9\omega t - 77.23° + 81.42°) + \ldots \propto$

The rms value of the third harmonic current is $5.68/\sqrt{2} = 4.02$ A

(b) Figure 11-29 shows the equivalent circuit for harmonic current. Using the current divider rule, the harmonic current through load is given by

$$\frac{I_h}{I_n} = \frac{X_C}{\sqrt{R^2 + (n\omega L - X_C)^2}}$$

where $X_c = 1/(n\,\omega\,C)$

For n = 3 and ω = 377, $\frac{I_h}{I_n} = \frac{X_C}{\sqrt{5^2 + (3 \times 3.77 - X_C)^2}} = 0.05 = 0.05$

which yields $X_c = -0.46389$ or 0.42704. Since X_c can not be negative, $X_c = 0.42704 = 1/(3 \times 377\,C)$ or C = 2070.5 μF